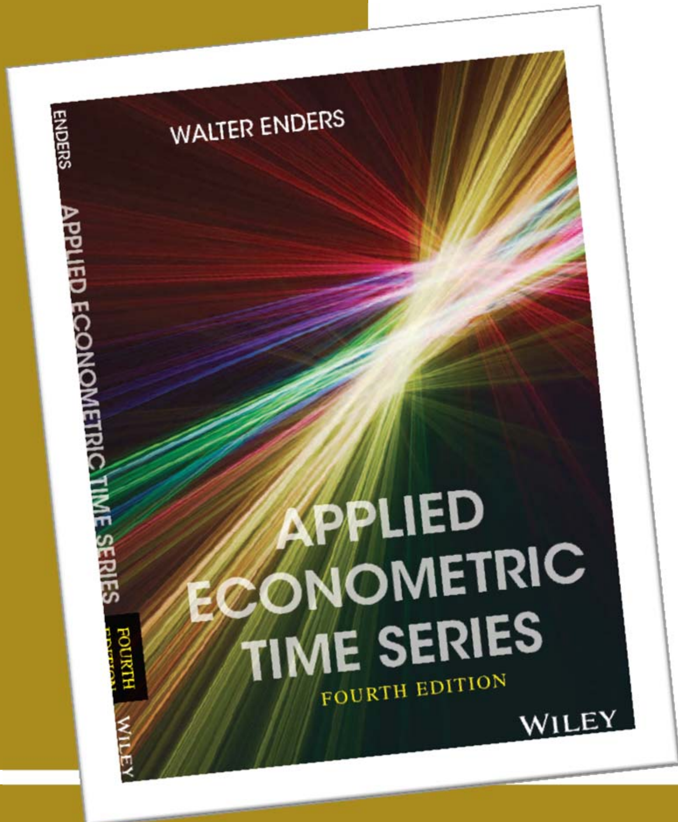

Applied Econometric Time Series 4th ed. Walter Enders

Chapter 5

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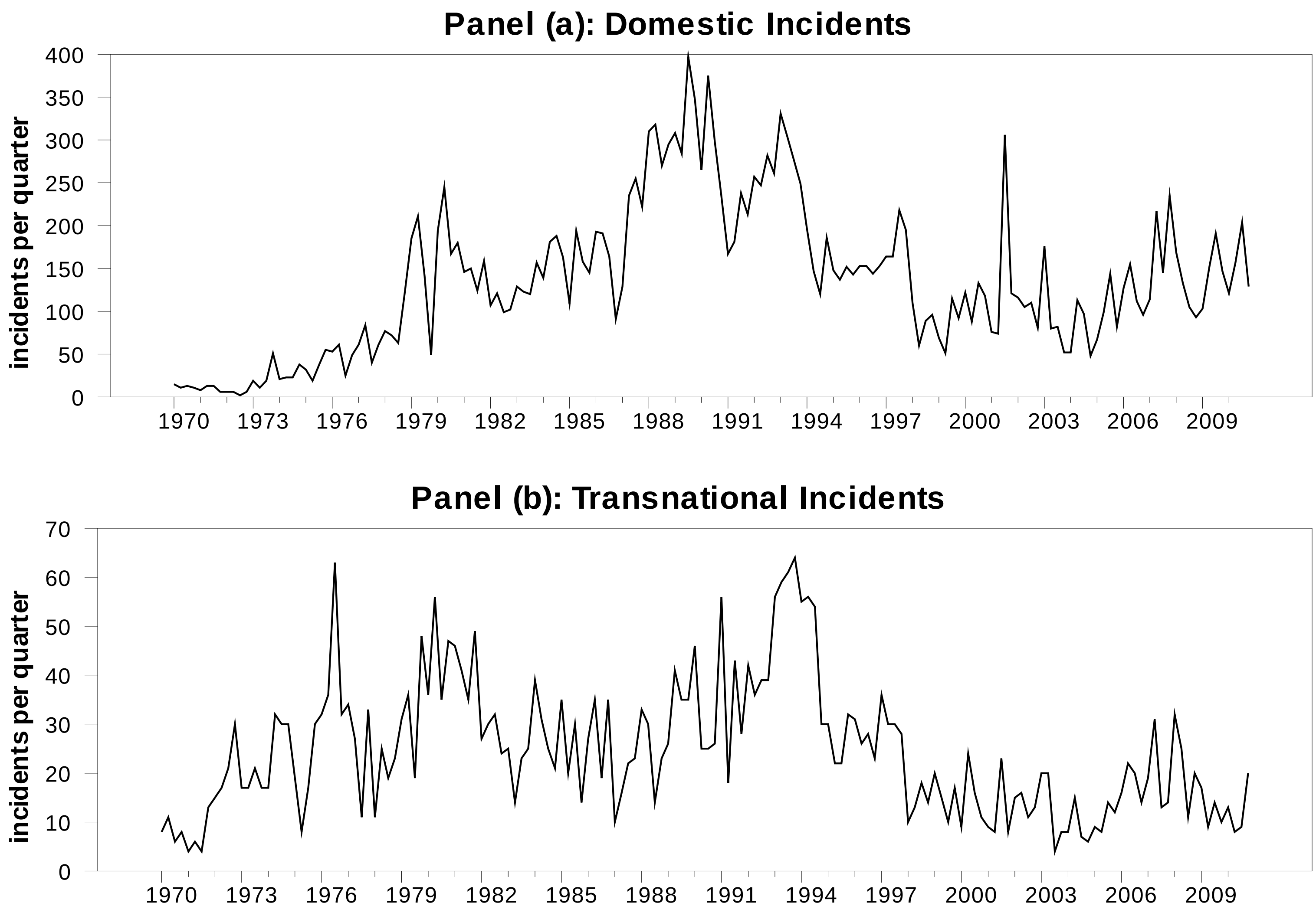


Figure 5.1 Domestic and Transnational Terrorism

An Intervention Model

Consider the model used in Enders, Sandler, and Cauley (1990) to study the impact of metal detector technology on the number of skyjacking incidents:

$$y_t = a_0 + a_1 y_{t-1} + c_0 z_t + \varepsilon_t, \qquad |a_1| < 1$$

where z_t is the intervention (or dummy) variable that takes on the value of zero prior to 1973Q1 and unity beginning in 1973Q1 and ε_t is a white-noise disturbance. In terms of the notation in Chapter 4, z_t is the level shift dummy variable D_L.

$$y_t = \frac{a_0}{1-a_1} + c_0 \sum_{i=0}^{\infty} a_1^i z_{t-i} + \sum_{i=0}^{\infty} a_1^i \varepsilon_{t-i}$$

Steps in an Intervention Model

- **STEP 1**: Use the longest data span (i.e., either the pre- or the postintervention observations) to find a plausible set of ARIMA models.
  - You can use the Perron (1989) test for structural change discussed in Chapter 4.
- **STEP 2**: Estimate the various models over the entire sample period, including the effect of the intervention.
- **STEP 3**: Perform diagnostic checks of the estimated equations.

Figure 5.2: Skyjackings

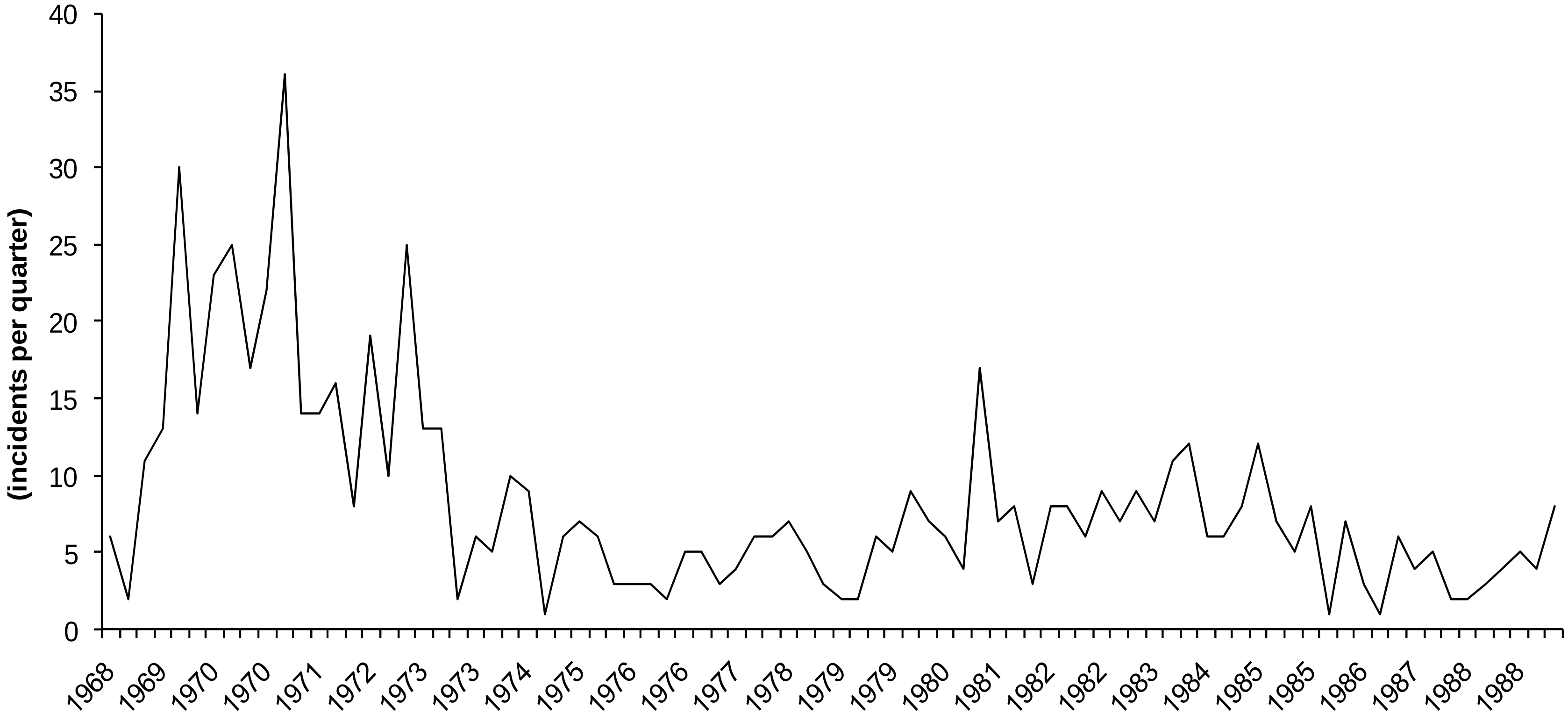


Figure 5.3: Typical Intervention Functions

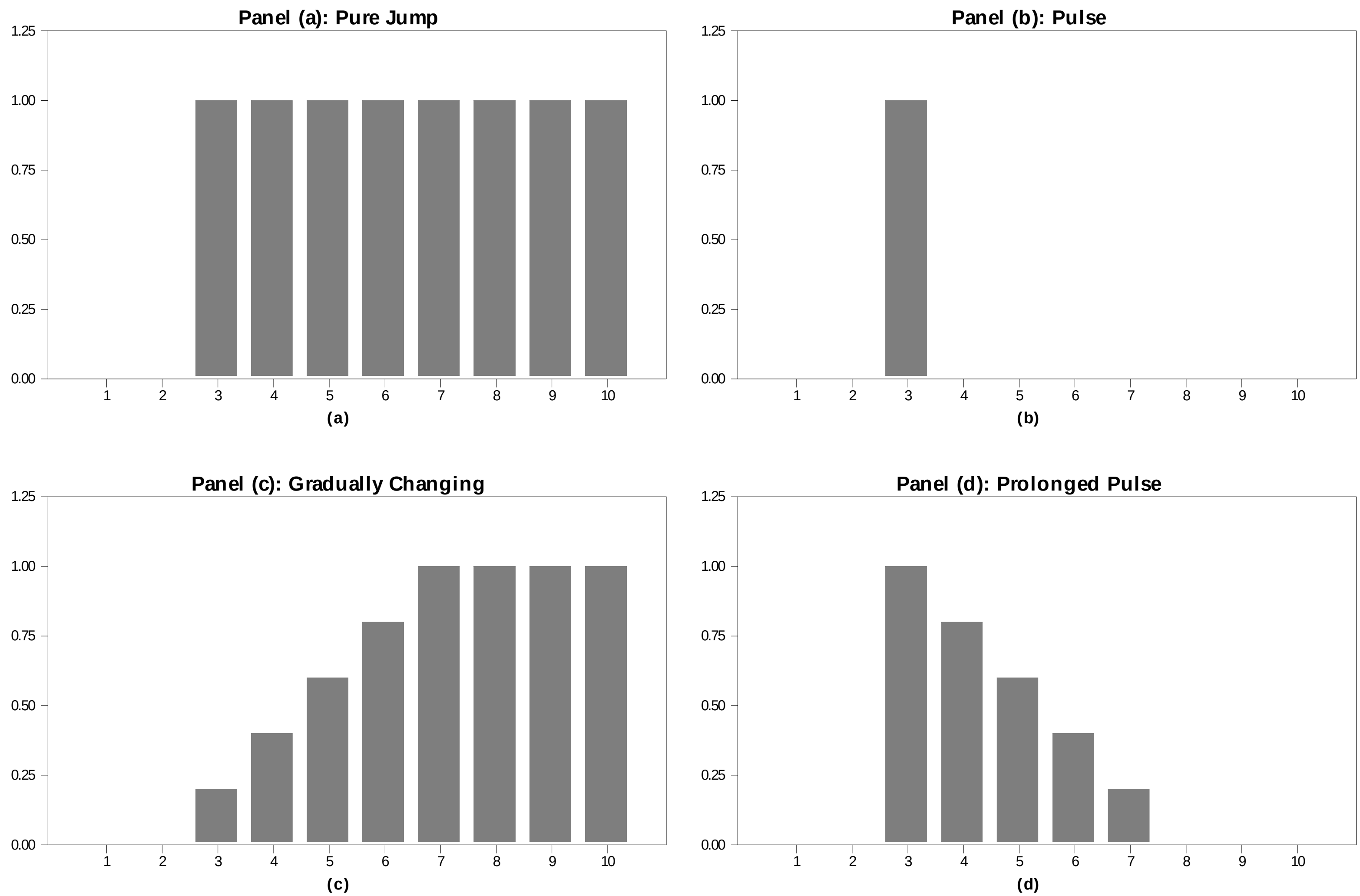


Table 5.1: Metal Detectors and Skyjackings

	Pre-Intervention Mean	a_1	Impact Effect (c_0)	Long-Run Effect
Transnational $\{TS_t\}$	3.032 (5.96)	0.276 (2.51)	−1.29 (-2.21)	−1.78
U.S. Domestic $\{DS_t\}$	6.70 (12.02)		−5.62 (−8.73)	−5.62
Other Skyjackings $\{OS_t\}$	6.80 (7.93)	0.237 (2.14)	−3.90 (−3.95)	−5.11

Notes:
1. *t*-statistics are in parentheses
2. The long-run effect is calculated as $c_0/(1 - a_1)$

ADLs and Transfer Functions

Transfer Functions

- $y_t = a_0 + A(L)y_{t-1} + C(L)z_t + B(L)\varepsilon_t$

where $A(L)$, $B(L)$, and $C(L)$ are polynomials in the lag operator L.

- In a typical transfer function analysis, the researcher will collect data on the endogenous variable $\{y_t\}$ and on the exogenous variable $\{z_t\}$. The goal is to estimate the parameter a_0 and the parameters of the polynomials $A(L)$, $B(L)$, and $C(L)$. Unlike an intervention model, $\{z_t\}$ is not constrained to have a particular deterministic time path.
- It is critical to note that transfer function analysis assumes that $\{z_t\}$ is an exogenous process that evolves independently of the $\{y_t\}$ sequence.

The CCVF

- The cross-correlation between y_t and z_{t-i} is defined to be

$$\rho_{yz}(i) \equiv \mathrm{cov}(y_t, z_{t-i})/(\sigma_y \sigma_z)$$

- where σ_y and σ_z = the standard deviations of y_t and z_t, respectively. The standard deviation of each sequence is assumed to be time independent.
- Plotting each value of $\rho_{yz}(i)$ yields the cross-correlation function (CCF) or **cross-correlogram.**

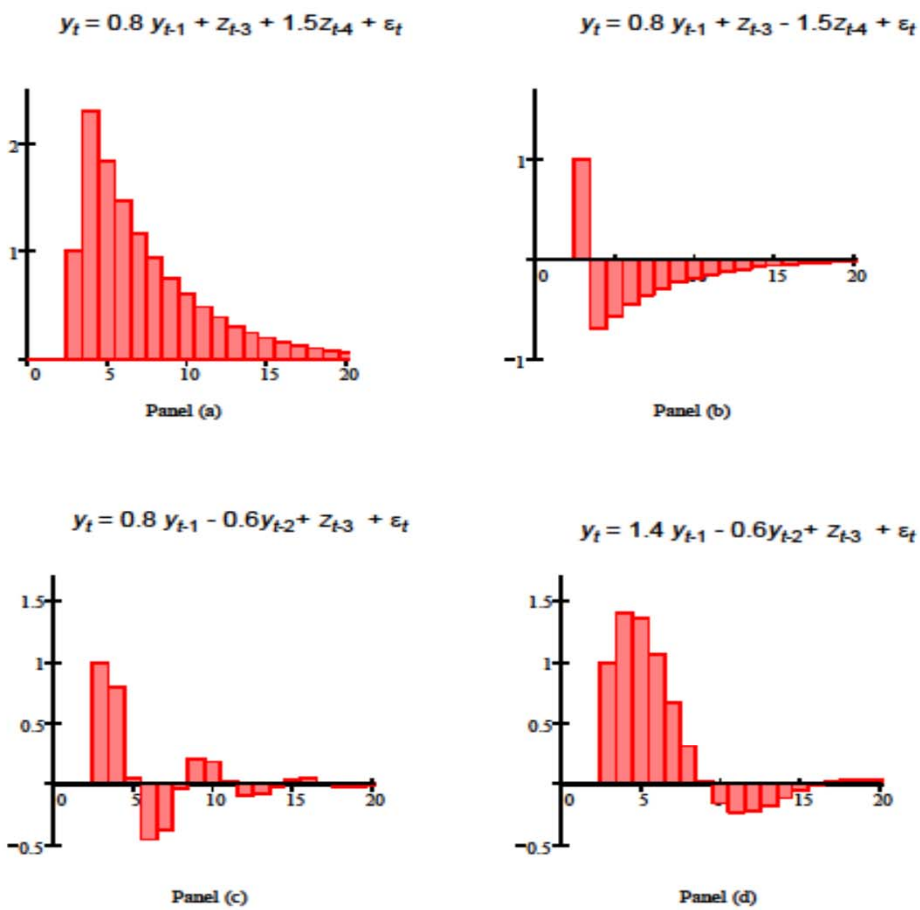
Interpreting the CCVF

- $y_t = a_0 + a_1 y_{t-1} + C(L)z_t + \varepsilon_t$ (5.7)

The theoretical CCVF (and CCF) has a shape with the following characteristics:

- All $\gamma_{yz}(i)$ will be zero until the first nonzero element of the polynomial $C(L)$.
- A spike in the CCVF indicates a nonzero element of $C(L)$. Thus, a spike at lag d indicates that z_{t-d} directly affects y_t.
- All spikes decay at the rate a_1; convergence implies that the absolute value of a_1 is less than unity. If $0 < a_1 < 1$, decay in the cross-covariances will be direct, whereas if $-1 < a_1 < 0$, the decay pattern will be oscillatory.
- Only the nature of the decay process changes if we generalize equation (5.7) to include additional lags of y_{t-i}.

Figure 5.4: Standardized Cross-Correlograms

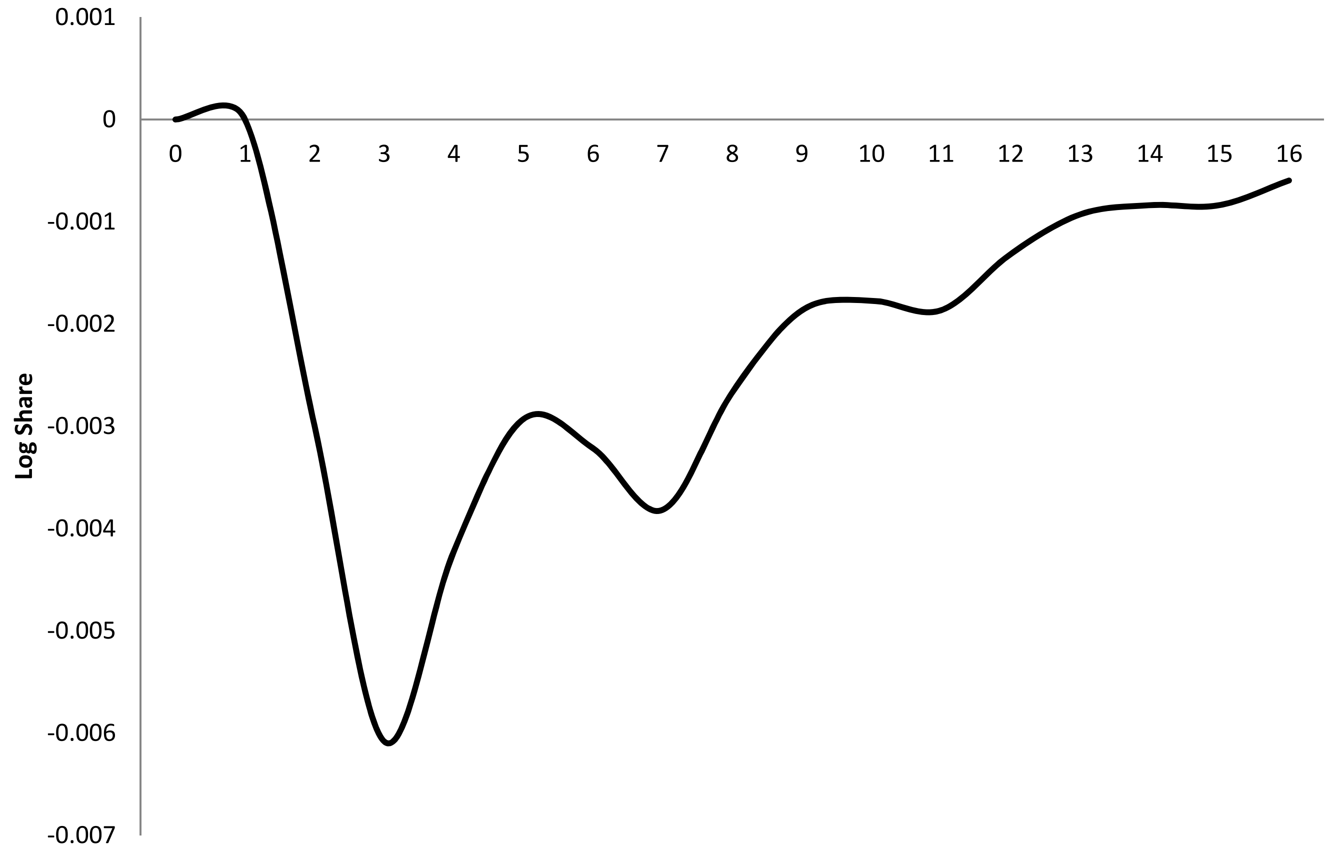


Estimating a Parsimonous ADL

- **STEP 1: Estimate the z_t sequence and an AR process.**
- **STEP 2: Identify plausible candidates for $C(L)$**
  - Constrict the filtered $\{y_t\}$ sequence by applying the filter $D(L)$ to each value of $\{y_t\}$; that is, use the results of Step 1 to obtain $D(L)y_t \equiv y_{ft}$.
- **STEP 3: Identify plausible candidates for the $A(L)$ function.** Regress y_t (not y_{ft}) on the selected values of $\{z_t\}$ to obtain a model of the form

  $$y_t = C(L)z_t + e_t$$
- **STEP 4:** Combine the results of Steps 2 and 3 to estimate the full equation. At this stage, you will estimate $A(L)$, and $C(L)$ simultaneously.

Figure 5.5 Italy's Share of Tourism



The Brookings Model

Consider two of the equations from the Brookings Quarterly Econometric Model

$$C_{NF} = 0.0656Y_D - 10.93[P_{CNF}/P_C]_{t-1} + 0.1889[N + N_{ML}]_{t-1}$$

(0.0165 (2.49) (0.0522)

$$C_{NEF} = 4.2712 + 0.1691Y_D - 0.0743[ALQD_{HH}/P_C]_{t-1}$$

(0.0127) (0.0213)

where: C_{NF} = personal consumption expenditures on food

Y_D = disposable personal income

P_{CNF} = price deflator for personal consumption expenditures on food

P_C = price deflator for personal consumption expenditures

N = civilian population

N_{ML} = military population including armed forces overseas

C_{NEF} = personal consumption expenditures for nondurables other than food

$ALQD_{HH}$ =end-of-quarter stock of liquid assets held by households

and: standard errors are in parenthesis.

The remaining portions of the model contain estimates for the other components of aggregate consumption, investment spending, government spending, exports, imports, for the financial sector, various price determination equations, …

Are such *ad hoc* behavioral assumptions consistent with economic theory? Sims (p.3, 1980) considers such multi-equation models and argues that:

"... what 'economic theory' tells us about them is mainly that any variable that appears on the right-hand-side of one of these equations belongs in principle on the right-hand-side of all of them. To the extent that models end up with very different sets of variables on the right-hand-side of these equations, they do so not by invoking economic theory, but (in the case of demand equations) by invoking an intuitive econometrician's version of psychological and sociological theory, since constraining utility functions is what is involved here. Furthermore, unless these sets of equations are considered as a system in the process of specification, the behavioral implications of the restrictions on all equations taken together may be less reasonable than the restrictions on any one equation taken by itself."

"St. Louis model" estimated by Anderson and Jordan (1968).

Using U.S. quarterly data from 1952 - 1968, they estimated the following reduced-form GNP determination equation:

$$\Delta Y_t = 2.28 + 1.54\Delta M_t + 1.56\Delta M_{t-1} + 1.44\Delta M_{t-2} + 1.29\Delta M_{t-3} + 0.40\Delta E_t + 0.54\Delta E_{t-1} - 0.03\Delta E_{t-2} - 0.74\Delta E_{t-3} \quad (5.16)$$

where ΔY_t = change in nominal GNP
ΔM_t = change in the monetary base
ΔE_t = change in "high employment" budget deficit

Testing whether the sum of the monetary base coefficients (i.e. 1.54 + 1.56 + 1.44 + 1.29 = 5.83) differs from zero yields a t-value of 7.25. Hence, they concluded that changes in the money base translate into changes in nominal GNP. On the other hand, the test that the sum of the fiscal coefficients (0.40 + 0.54 - 0.03 - 0.74 = 0.17) equals zero yields a t-value of 0.54. According to Anderson and Jordan, the results support "lagged crowding out" in the sense that an increase in the budget deficit initially stimulates the economy.

Reduced Form

Sims (1980) also points out several problems with this type of analysis.

Ensuring that there is no feedback between GNP and the money base or the budget deficit. However, the assumption of no feedback is unreasonable if the monetary or fiscal authorities deliberately attempt to alter nominal GNP. As in the thermostat example, if the monetary authority attempts to control the economy by changing the money base, we can not identify the "true" model. In the jargon of time-series econometrics, changes in GNP would "cause" changes in the money supply. One appropriate strategy would be to simultaneously estimate the GNP determination equation *and* the money supply feedback rule.

Comparing the two types of models, Sims (pp. 14-15, 1980) states:
"Because existing large models contain too many incredible restrictions, empirical research aimed at testing competing macroeconomic theories too often proceeds in a single- or few- equation framework. For this reason alone, it appears worthwhile to investigate the possibility of building large models in a style which does not tend to accumulate restrictions so haphazardly. ... It should be feasible to estimate large-scale macromodels as unrestricted reduced forms, treating all variables as endogenous."

Structural VARs

$$y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t\text{-}1} + \gamma_{12}z_{t\text{-}1} + \varepsilon_{yt}$$
$$z_t = b_{20} - b_{21}y_t + \gamma_{21}y_{t\text{-}1} + \gamma_{22}z_{t\text{-}1} + \varepsilon_{zt}$$

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}\begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t\text{-}1} + \varepsilon_t$$

Pre-multiply by B^{-1} to obtain

$$x_t = A_0 + A_1 x_{t\text{-}1} + e_t$$

$$A_0 = B^{-1}\Gamma_0; A_1 = B^{-1}\Gamma_1; \text{ and } e_t = B^{-1}\varepsilon_t$$

A 1st-Order VAR in Standard Form

$$y_t = a_{10} + a_{11}y_{t\text{-}1} + a_{12}z_{t\text{-}1} + e_{1t}$$

$$z_t = a_{20} + a_{21}y_{t\text{-}1} + a_{22}z_{t\text{-}1} + e_{2t}$$

$$e_{1t} = (\varepsilon_{yt} - b_{12}\varepsilon_{zt})/(1 - b_{12}b_{21})$$
$$e_{2t} = (\varepsilon_{zt} - b_{21}\varepsilon_{yt})/(1 - b_{12}b_{21})$$

The VAR Structure

Consider the following 2-variable 1-lag VAR in standard form:

$$y_t = a_{10} + a_{11}y_{t-1} + a_{12}z_{t-1} + e_{1t}$$

$$z_t = a_{20} + a_{21}y_{t-1} + a_{22}z_{t-1} + e_{2t}$$

It is assumed that e_{1t} and e_{2t} are serially uncorrelated but the covariance $Ee_{t1}e_{2t}$ need not be zero. If the variances and covariance are time-invariant, we can write the variance/covariance matrix as:

where: Var(e_{it}) = σ_{ii} and Cov(e_{1t},e_{2t}) = $\sigma_{12} = \sigma_{21}$.

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

Forecasting

- If your data run through period T, it is straightforward to obtain the one-step-ahead forecasts of your variables using the relationship
- $E_T x_{T+1} = A_0 + A_1 x_T$.
- A two-step-ahead forecast can be obtained recursively from $E_T x_{T+2} = A_0 + A_1 E_T x_{T+1} = A_0 + A_1[A_0 + A_1 x_T]$.
- Since unrestricted VARs are overparameterized, the forecasts may be unreliable. In order to obtain a parsimonious model, many forecasters would purge the insignificant coefficients from the VAR.
- After reestimating the so-called **near-VAR** model using *SUR*, it could be used for forecasting purposes.

Terrorism and Israeli real per capita GDP

$$\begin{bmatrix} \Delta GDP_t \\ \Delta I_t \\ \Delta EXP_t \\ \Delta NDC_t \end{bmatrix} = \begin{bmatrix} A_{11}(L) & \dots & A_{14}(L) \\ \vdots & \ddots & \vdots \\ A_{41}(L) & \dots & A_{44}(L) \end{bmatrix} \begin{bmatrix} \Delta GDP_{t-1} \\ \Delta I_{t-1} \\ \Delta EXP_{t-1} \\ \Delta NDC_{t-1} \end{bmatrix} + \begin{bmatrix} c_1 T_{t-1} \\ c_2 T_{t-1} \\ c_3 T_{t-1} \\ c_3 T_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix}$$

The aim of the study was to investigate the effects of terrorism (T) on the growth rates of Israeli real per capita GDP (ΔGDP_t), investment (ΔI_t), exports (ΔEXP_t), and nondurable consumption (ΔNDC_t). The authors use quarterly data running from 1980Q1 to 2003Q3 so that there are 95 total observations.

Cost of terrorism

- To forecast the values of x_{T+2} and beyond, it is necessary to know the magnitude of the terrorism variable over the forecast period. Toward this end, they supposed that all terrorism actually ended in 2003Q4 (so that all values of $T_j = 0$ for $j >$ 2003Q4). Under this assumption, the annual growth rate of *GDP* was estimated to be 2.5% through 2005Q3. Instead, when they set the values of T_j at the 2000Q4 to 2003Q4 period average, the growth rate of GDP was estimated to be zero. Thus, a steady level of terrorism would have cost the Israeli economy all of its real output gains. In actuality, the largest influence of terrorism was found to be on investment. The impact of terrorism on investment was twice as large as the impact on real GDP.

Impulse Responses

Consider a 2-variable model:

$$y_t = \sum_{i=1}^{n} a_{11}(i) y_{t-i} + \sum_{i=1}^{n} a_{12}(i) z_{t-i} + e_{1t}$$

$$z_t = \sum_{i=1}^{n} a_{21}(i) y_{t-i} + \sum_{i=1}^{n} a_{22}(i) z_{t-i} + e_{2t}$$

The impulse response function is obtained using the moving average representation:

$$y_t = \sum_{i=1}^{n} b_{11}(i) e_{1t-i} + \sum_{i=1}^{n} b_{12}(i) e_{2t-i} + e_{1t}$$

$$z_t = \sum_{i=1}^{n} b_{21}(i) e_{1t-i} + \sum_{i=1}^{n} b_{22}(i) e_{2t-i} + e_{2t}$$

Impulse Responses: An Example

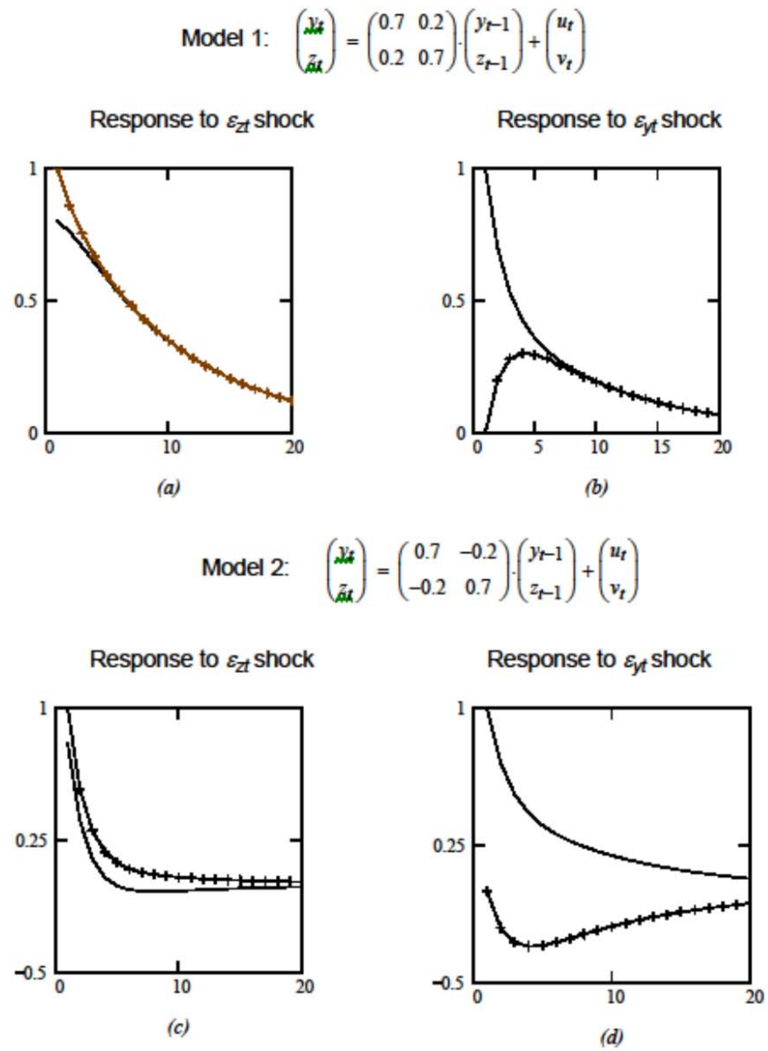
x(t) = 0.7*x(t-1) + 0.2*y(t-1) + e1(t)

y(t) = 0.2*x(t-1) + 0.7y(t-1) + e2(t)

e2(t) = 0.2*e1(t)

	1-unit e1 shock				1-unit e2 shoc	
t	**x(t)**	**y(t)**		**t**	**x(t)**	**y(t)**
1	1	0.2		1	0	1
2	0.74	0.34		2	0.2	0.7
3	0.586	0.386		3	0.28	0.53
4	0.487	0.387		4	0.302	0.427
5	0.419	0.369		5	0.297	0.359
6	0.367	0.342		6	0.28	0.311
7	0.325	0.313		7	0.258	0.274
8	0.29	0.284		8	0.235	0.243
9	0.26	0.257		9	0.213	0.217
10	0.233	0.232		10	0.193	0.195
11	0.21	0.209		11	0.174	0.175
12	0.188	0.188		12	0.157	0.157
13	0.17	0.169		13	0.141	0.141
14	0.153	0.152		14	0.127	0.127

Figure 5.7: Two Impulse Response Functions



Legend: Solid line = $\{y_t\}$ sequence Cross-hatch = $\{z_t\}$ sequence

Note: In all cases $u_t = 0.8v_t + \varepsilon_{yt}$ and $v_t = \varepsilon_{zt}$

The Residuals vs the Pure Shocks

$e_{1t} = (\varepsilon_{yt} - b_{12}\varepsilon_{zt})/(1-b_{12}b_{21})$
$e_{2t} = (\varepsilon_{zt} - b_{21}\varepsilon_{yt})/(1-b_{12}b_{21})$

If we set b_{12} or b_{21} equal to zero, we can identify the shocks

Identification

$$e_{1t} = g_{11}\varepsilon_{1t} + g_{12}\varepsilon_{2t}$$
$$e_{2t} = g_{21}\varepsilon_{1t} + g_{22}\varepsilon_{2t}$$

or:

$$e_t = G\varepsilon_t$$

If we let $var(\varepsilon_{1t}) = \sigma_1^2$ and $var(\varepsilon_{2t}) = \sigma_2^2$, it follows that:

$$E\varepsilon_{1t}\varepsilon_{2t} \equiv \Sigma_\varepsilon = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}$$

The problem is to identify the unobserved values of ε_{1t} and ε_{2t} from the regression residuals e_{1t} and e_{2t}.

Identification 2

If we knew the four values g_{11}, g_{12} g_{13} and g_{14} we could obtain all of the structural shocks for the regression residuals. Of course, we do have some information about the values of the g_{ij}. Consider the variance/covariance matrix of the regression residuals:

$$Eee' = \Sigma \qquad \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

Sim's Recursive Ordering

Sim's **recursive ordering** restricts on the primitive system such that the coefficient b_{21} is equal to zero. Writing (5.17) and (5.18) with the constraint imposed yields

$$y_t = b_{10} - b_{12}z_t + g_{11}y_{t-1} + g_{12}z_{t-1} + e_{yt}$$

$$z_t = b_{20} + g_{21}y_{t-1} + g_{22}z_{t-1} + e_{zt}$$

Similarly, we can rewrite the relationship between the pure shocks and the regression residuals given by (5.22) and (5.23) as

$$e_{1t} = \varepsilon_{yt} - b_{12}\varepsilon_{zt}$$
$$e_{2t} = \varepsilon_{zt}$$

Sims' Recursive Ordering

$$e_{1t} = \varepsilon_{yt} - b_{12}\varepsilon_{zt}$$
$$e_{2t} = \varepsilon_{zt}$$

so that

$$\text{var}(e_1) = \sigma_y^2 + b_{12}^2\sigma_z^2 \qquad (5.31)$$

$$\text{var}(e_2) = \sigma_z^2 \qquad (5.32)$$

$$\text{cov}(e_1, e_2) = -b_{12}\sigma_z^2 \qquad (5.33)$$

Hence, it must be the case that:

$$Ee_te_t' = EG\varepsilon_t\varepsilon_t'G'$$

Since $Ee_te_t' = \Sigma$ and $E\varepsilon_t\varepsilon_t' = \Sigma_\varepsilon$, it follows that:

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = G\Sigma_\varepsilon G'$$

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} g_{11}{}^2 + g_{12}{}^2 & g_{11}g_{21} + g_{12}g_{22} \\ g_{11}g_{21} + g_{12}g_{22} & g_{21}{}^2 + g_{22}{}^2 \end{pmatrix}$$

In general you must fix $(n^2 - n)/2$ elements for exact identification

Hypothesis Tests

Let Σ_u and Σ_r be the variance/covariance matrices of the unrestricted and restricted systems, respectively. Then, in large samples:

$$(T\text{-}c)(\log|\Sigma_r| - \log|\Sigma_u|)$$

can be compared to a χ^2 distribution with degrees of freedom equal to the number of restrictions.

Model Selection Criteria

Alternative test criteria are the multivariate generalizations of the AIC and SBC:

$$\text{AIC} = T \log |\Sigma| + 2N$$
$$\text{SBC} = T \log |\Sigma| + N \log(T)$$

Where $|\Sigma|$ = determinant of the variance/covariance matrix of the residuals and N = total number of parameters estimated *in all equations*.

Granger-Causality

Granger causality: If $\{y_t\}$ does not improve the forecasting performance of $\{z_t\}$, then $\{y_t\}$ does not Granger-cause $\{z_t\}$. The practical way to determine Granger causality is to consider whether the lags of one variable enter into the equation for another variable.

Block Exogeneity

Block exogeneity restricts all lags of w_t in the y_t and z_t equations to be equal to zero. This cross-equation restriction is properly tested using the likelihood ratio test. Estimate the y_t and z_t equations using lagged values of $\{y_t\}$, $\{z_t\}$, and $\{w_t\}$ and calculate Σ_u. Reestimate excluding the lagged values of $\{w_t\}$ and calculate Σ_r. Form the likelihood ratio statistic:

$$(T-c)(\log | \Sigma_r | - \log | \Sigma_u |$$

This statistic has a chi-square distribution with degrees of freedom equal to $2p$ (since p lagged values of $\{w_t\}$ are excluded from each equation). Here $c = 3p + 1$ since the unrestricted y_t and z_t equations contain p lags of $\{y_t\}$, $\{z_t\}$, and $\{w_t)$ plus a constant.

To Difference or Not to Difference

- Recall a key finding of Sims, Stock, and Watson (1990):

  If the coefficient of interest can be written as a coefficient on a stationary variable, then a t-test is appropriate.
- You can use t-tests or F-tests on the stationary variables.
- You can perform a lag length test on any variable or any set of variables
- Generally, you cannot use Granger causality tests concerning the effects of a nonstationary variable
- The issue of differencing is important.
  - If the VAR can be written entirely in first differences, hypothesis tests can be performed on any equation or any set of equations using t-tests or F-tests.
  - It is possible to write the VAR in first differences if the variables are $I(1)$ and are *not* cointegrated. If the variables in question are cointegrated, the VAR cannot be written in first differences

If the $I(1)$ variables are not cointegrated and you use levels:

- Tests lose power because you estimate n^2 more parameters (one extra lag of each variable in each equation).
- For a VAR in levels, tests for Granger causality conducted on the $I(1)$ variables do not have a standard F distribution. If you use first differences, you can use the standard F distribution to test for Granger causality.
- When the VAR has $I(1)$ variables, the impulse responses at long forecast horizons are inconsistent estimates of the true responses. Since the impulse responses need not decay, any imprecision in the coefficient estimates will have a permanent effect on the impulse responses. If the VAR is estimated in first differences, the impulse responses decay to zero and so the estimated responses are consistent.

Seemingly Unrelated Regressions

Different lag lengths

$$y_t = a_{11}(1)y_{t-1} + a_{11}(2)y_{t-2} + a_{12}z_{t-1} + e_{1t}$$
$$z_t = a_{21}y_{t-1} + a_{22}z_{t-1} + e_{2t}$$

Non-Causality

$$y_t = a_{11}y_{t-1} + e_{1t}$$
$$z_t = a_{21}y_{t-1} + a_{22}z_{t-1} + e_{2t}$$

Effects of a third variable

$$y_t = a_{11}y_{t-1} + a_{12}z_{t-1} + e_{1t}$$
$$z_t = a_{21}y_{t-1} + a_{22}z_{t-1} + a_{23}w_t + e_{2t}$$

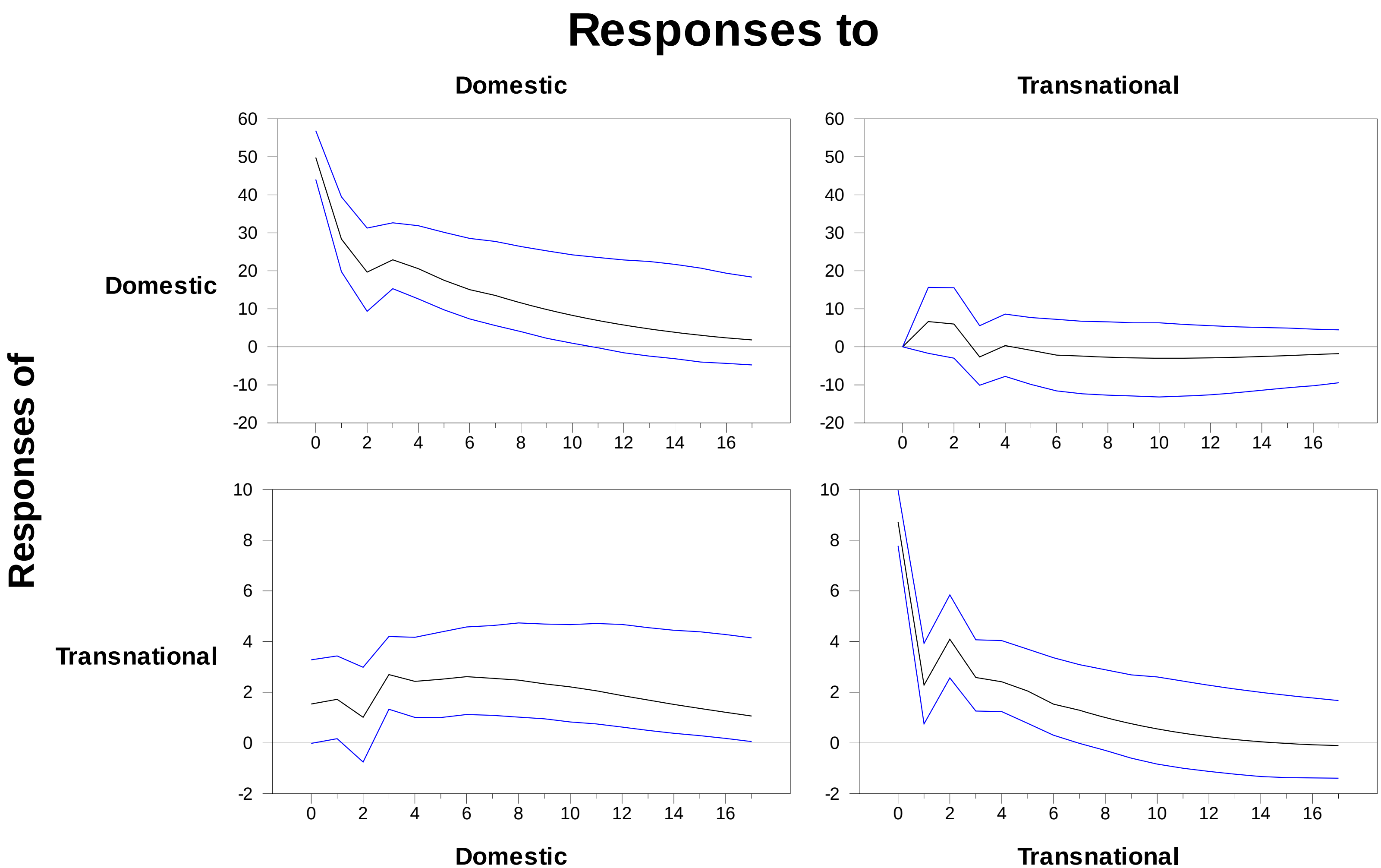


Figure 5.8 Impulse Responses of Terrorism

Sims Bernamke

$$\begin{bmatrix} e_{yt} \\ e_{mt} \\ e_{rt} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ g_{21} & 1 & g_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{mt} \\ \varepsilon_{rt} \end{bmatrix}$$

Sims' Structural VAR

Sims (1986) used a six-variable VAR of quarterly data over the period 1948*Q*1 to 1979*Q*3. The variables included in the study are real GNP (y), real business fixed investment (i), the GNP deflator (p), the money supply as measured by M1 (m), unemployment (u), and the treasury bill rate (r).

$$\begin{bmatrix} 1 & b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & 1 & b_{23} & b_{24} & 0 & 0 \\ b_{31} & 0 & 1 & 0 & 0 & b_{36} \\ b_{41} & 0 & b_{43} & 1 & 0 & b_{46} \\ b_{51} & 0 & b_{53} & b_{54} & 1 & b_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_t \\ m_t \\ y_t \\ p_t \\ u_t \\ i_t \end{bmatrix} = \begin{bmatrix} \varepsilon_{rt} \\ \varepsilon_{mt} \\ \varepsilon_{yt} \\ \varepsilon_{pt} \\ \varepsilon_{ut} \\ \varepsilon_{it} \end{bmatrix}$$

Note that it is Overidentified

$r_t = 71.20m_t + e_{rt}$ (5.59)

$m_t = 0.283y_t + 0.224p_t - 0.0081r_t + e_{mt}$ (5.60)

$y_t = -0.00135r_t + 0.132i_t + e_{yt}$ (5.61)

$p_t = -0.0010r_t + 0.045y_t - 0.00364i_t + e_{pt}$ (5.62)

$u_t = -0.116r_t - 20.1y_t - 1.48i_t - 8.98p_t + e_{ut}$ (5.63)

$i_t = e_{it}$ (5.64)

Sims views (5.59) and (5.60) as money supply and demand functions, respectively. In (5.59), the money supply rises as the interest rate increases. The demand for money in (5.60) is positively related to income and the price level and negatively related to the interest rate. Investment innovations in (5.64) are completely autonomous. Otherwise, Sims sees no reason to restrict the other equations in any particular fashion. For simplicity, he chooses a Choleski-type block structure for GNP, the price level, and the unemployment rate. The impulse response functions appear to be consistent with the notion that money supply shocks affect prices, income, and the interest rate.

Blanchard-Quah

Suppose we are interested in decomposing an $I(1)$ sequence, say $\{y_t\}$, into its temporary and permanent components. Let there be a second variable $\{z_t\}$ that is affected by the same two shocks. The BMA representation is:

$$\begin{bmatrix} \Delta y_t \\ z_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

$$\Sigma_\varepsilon = \begin{bmatrix} \operatorname{var}(\varepsilon_1) & \operatorname{cov}(\varepsilon_1, \varepsilon_2) \\ \operatorname{cov}(\varepsilon_1, \varepsilon_2) & \operatorname{var}(\varepsilon_2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The Long-run resrtiction

- Assume that one of the shocks has a temporary effect on the $\{y_t\}$ sequence.
  - It is this dichotomy between temporary and permanent effects that allows for the complete identification of the structural innovations from an estimated VAR.
- For example, Blanchard and Quah assume that an aggregate demand shock has no long-run effect on real GNP. In the long run, if real GNP is to be unaffected by the demand shock, it must be the case that the cumulated effect of an ε_{1t} shock on the Δy_t sequence must be equal to zero. Hence, the coefficients $c_{11}(k)$ must be such that

$$\sum_{k=0}^{\infty} c_{11}(k)\varepsilon_{1t-k} = 0$$

Since this must be true for all realizations

$$\sum_{k=0}^{\infty} c_{11}(k) = 0$$

Recall that:

$$e_{1t} = c_{11}(0)e_{1t} + c_{12}(0)e_{2t}$$

$$e_{2t} = c_{21}(0)e_{1t} + c_{22}(0)e_{2t}$$

The four restrictions

- Restriction 1:

$$\mathrm{var}(e_1) = c_{11}(0)^2 + c_{12}(0)^2$$

Restriction 2:

$$\mathrm{var}(e_2) = c_{21}(0)^2 + c_{22}(0)^2$$

Restriction 3:

$$Ee_{1t}e_{2t} = c_{11}(0)c_{21}(0) + c_{12}(0)c_{22}(0)$$

Blanchard-Quah

Changes in ε_{1t} will have no long-run effect on the $\{y_t\}$ sequence if:

$$\left[1-\sum_{k=0}^{\infty} a_{22}(k)\right] c_{11}(0)+\sum_{k=0}^{\infty} a_{12}(k) c_{21}(0)=0$$

Forecast Error Variance Due to Demand-side Shocks

Horizon	Output	Unemployment
1	99.0	51.9
4	97.9	80.2
12	67.6	86.2
40	39.3	85.6

Figure 5.9 Responses of Real and Nominal Exchange Rates