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**Pretesting For Multi-Step Ahead Forecasts with STAR Models**

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**Abstract**

It is well known that a linear model may forecast better than a nonlinear one, even when the nonlinear model is consistent with the actual data-generating process. Moreover, forecasting with nonlinear models can be quite programming-intensive as multistep-ahead forecasts need to be simulated. We propose a simple pretest to help determine whether it is worthwhile to forecast a series using a *STAR* model. In particular, we extend Teräsvirta’s in-sample test for *LSTAR* and *ESTAR* behavior to multistep-ahead out-of-sample forecasts. We apply our pretest to the real exchange rates of various OECD countries. When the test strongly rejects the null of linearity, a nonlinear model clearly outperforms a linear one in terms of multistep-ahead forecasting accuracy (i.e., lower mean absolute percentage error). However, when it fails to reject the null or it does so only mildly, a direct approach based on an autoregressive model yields forecasts that are slightly superior to those forecasts generated from a logistic model. We also find that when the proposed test strongly rejects the null of linearity, the “direct” method of forecasting and the bootstrap predictor yield a similar performance with the latter outperforming in terms of a lower mean absolute percentage error.

**Keywords**: Nonlinear Models, Forecasting, Linearity Testing

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\*Corresponding author.**1. Introduction**

Teräsvirta (2006) summarizes much of the research indicating that a linear model may forecast better than a nonlinear one, even when the nonlinear model is consistent with the actual data-generating process. For example, Montgomery *et al*. (1998) show that a nonlinear model may forecast better than a linear one in some regimes but not in others (e.g., recessions but not expansions). Similarly, Dacco and Satchell (1999) show that a regime-switching model may have poor forecasting performance relative to a linear model as a result of misclassifying observations. The point is that, any superior in-sample performance of a nonlinear model may not translate itself into superior out-of-sample performance.

 White’s (2006) survey article details the “potentially serious practical challenges” inherent in using a nonlinear model to make multistep-ahead forecasts. He shows how such forecasts are computationally difficult to program, entail the possibility of overfitting, and are often difficult to interpret. In the same volume, Teräsvirta (2006) highlights the different methodological choices that can lead a researcher astray when using a nonlinear model to make multi-step ahead forecasts. For example, forecasters need to select the appropriate model, decide whether to use direct or indirect methods, decide whether to use Monte Carlo or bootstrapped forecasts, and select the appropriate histories for their simulations. As evidenced by time-series forecasting competitions (see Weigend and Gershenfeld (1994)), different well-trained researchers using the same data set will often come up with very different multi-step ahead forecasts. In order to avoid many of these forecasting issues, we extend the nonlinearity tests of Luukkonen (1988), Tsay (1991), and Teräsvirta (1994). Specifically, we extrapolate their in-sample methodologies to develop a simple pretest to determine whether it is worthwhile to forecast a series using a smooth transition autoregressive (*STAR*) model or a linear model.

We focus our attention on *STAR* models because they are particularly well-suited to characterize important macroeconomic variables such as the unemployment rate, the growth rate of industrial production, and exchange rates (see Stock and Watson (1998) and Marcellino (2002, 2004) among others). Moreover, as will be shown, the multistep-ahead forecasts of the type of STAR models we consider have a STAR form. This greatly eases the specification problem in that the form of the model does not change at various forecasting horizons. Nevertheless, testing for linearity directly is not straightforward because of the unidentified parameters under the null (see Davis (1987)). However, in the context of *STAR* models, Teräsvirta (1994) shows how to circumvent this nuisance parameter problem. The solution consists in taking a Taylor series approximation of the nonlinear component (i.e. the transition function) of the *STAR* process. The paper adopts this approach with the aim of establishing whether the data-generating process appears nonlinear at several steps ahead.

Several papers, including Michael, Nobay, and Peel (1997), Leybourne *et* *al*. (1998), Sollis *et* *al*. (2002), Kapetanios *et al*. (2003), Kilian and Taylor (2003) and Bec *et al*. (2004), suggest that real exchange rate behavior can be characterized be some type of nonlinear adjustment. As such, it seems natural to apply our test to a set of real exchange rates from a variety of OECD countries. When our linearity test strongly rejects the null of a linear model, forecasts from a nonlinear model have lower mean square prediction errors (*MSPEs*)than forecasts from a linear model. Moreover, when the linearity test fails to reject the null of a linear model, forecasts from the *STAR* model and its linear counterpart display similar *MSPEs*. We show that the nonlinear exchange rate models can also beat the benchmark random-walk forecasts.

The remainder of the paper is organized as follows. Section 2 serves as a brief literature review in that it describes the logistic (*LSTAR*) and exponential(*ESTAR*) forms of the *STAR* model. Section 3 shows how we can generalize Teräsvirta’s (1994) test for STAR behavior to multi-step ahead forecasts. Section 4 performs several size, and power exercises to assess the small sample properties of our pretest. Section 5 applies the test to the real exchange rates of many OECD countries and uses the results of the test to construct out-of-sample forecasts. We follow the lead of Lin and Granger (1994) and construct various out-of-sample forecasts using both the direct and iterative (recursive) forecasting methods. We find that the direct method has the advantage that no numerical integration and/or approximation is necessary and the forecasts can be produced as in the one‐step‐ahead case. When our test indicates that the data-generating process (*DGP*) is nonlinear, it turns out that on average iterated forecasts from a nonlinear model of the real exchange rates have a lower *MAPE* than those from a direct approach. In contrast, when our test indicates that the *DGP* is linear, the direct method results in a lower root mean squared error. Section 6 summarizes the key results.

# 2. The Smooth Transition Regression Model

As detailed in Granger and Teräsvirta (1993) and Teräsvirta (2006), a univariate smooth transition autoregressive (*STAR*) model has the following representation:

 *yt*+1 = *αwt* + *βwtG*(*θ*; *yt−d*; *c*) + *εt*+1 , *t* = 1, …,*T* (1)

*where* *εt*+1 ~ *iid*(0, *σ*2 ), *α*, *β*, and *c*, are unknown parameters, *wt* = (*yt*, …,*yt−p*), *d* ≥ 0 is the delay parameter, and the transition function *G*(*θ*; *yt−d*; *c*) is continuous and may be either odd or even.

One common formulation of the transition function is the logistic equation:

 *G*(*θ*; *yt−d*; *c*) = [1 + exp(−*θ*(*yt−d* − *c*))]−1 (2)

*where* *c* is the location (or threshold) parameter.

In (2), the *θ* parameter represents the slope of the transition function *G*(*θ*; *yt−d*; *c*). Specifically, when *θ* = 0 the transition function *G*(*θ*; *yt−d*; *c*) = 1/2 so that the *LSTAR* model nests a linear model. Conversely, as | *θ* | → ∞, the *LSTAR* model approaches a threshold autoregressive model with two distinct regimes. The transition function is a bounded function of the transition variable *yt−d* and is continuous everywhere in the parameter space for any value of *yt−d*. Given that *G*(*θ*; *yt−d*; *c*) is odd and monotonically increasing, whenever | *yt−d − c* | is large and *yt−d* < *c*, *yt*+1 is effectively generated by the linear model:

 *yt*+1 = *αwt* + *εt*+1, *t* = 1, …, *T* (3)

If | *yt−d − c* | is large and *yt−d* < *c*, *yt*+1 is effectively generated by:

 *yt*+1 = (*α + β*)*wt* + *εt*+1, *t* = 1, …, *T* (4)(3) (4)

Thus, the parameters of the autoregressive process change monotonically as a function of *yt−d*. Such a model might be appropriate for real exchange rate behavior if the speed of adjustment depends on whether the rate is appreciating or depreciating.

If *G*(*θ*; *yt−d*; *c*) is assumed even, it is standard to use the exponential function to model the transition between two regimes. The *ESTAR* transition function can be written as:

 (5)

The parameters *θ*, *c*, and *d* are defined as above. Similar to the logistic case, the exponential function is bounded between 0 and 1, cases that correspond to *θ* = 0 and | *θ* | → ∞, respectively. Equation (1) with (5) has been employed especially to test the empirical validity of the purchasing power parity (*PPP*) hypothesis (see Taylor and Sarno (2002) for a review).

## *Forecasting with a STAR Process*

Forecasting with a linear model is straightforward. For example, if one desires to obtain forecasts of *yt*+*h* using information available up to *t* then obtaining an iterated (or recursive) forecast entails the following:

* estimate a linear model using the information available up to time period *t*;
* next, feed the forward forecasts of the early periods for use in the later periods up to step *h* using the one‐period ahead model. Given that the model for *yt*+1 is given by:

  (6)

the one-step-ahead forecast is



and the iterated forecasts are:

  (7)

 In contrast, the direct forecasting method simply requires one to regress the *h*‐step ahead value of *yt+h* directly on *yt* through *yt−p* to obtain:

  (8)

*where * denote the parameters estimated using the direct method. Obviously, except for the one-step ahead forecast, the parameters of (7) will differ from those of (8). Moreover, the parameters of (8) will generally be a function of the forecast horizon *h*.

Unfortunately, forecasting recursively more than one period ahead using a nonlinear model like the *STAR* requires numerical techniques. The difficulty follows from the fact that *G*(*θ*; *yt−d*; *c*) is nonlinear so that for *h* > *d*,

 *EtG*(*θ*; *yt-d+h*; *c*) ≠ *G*(*θ*; *Etyt-d+h*; *c*). (9)

Hence, it is not possible to plug the forecasts of *Etyt*-*d+h* into the transition function to obtain multistep ahead forecasts. Instead, as detailed in Teräsvirta (2006) and Enders (2010), multistep ahead forecasts can be can be made using a Monte Carlo simulation of the residuals (drawn from a standard normal distribution) or by bootstrapping the residuals.

 Since there are likely to be specification errors in constructing the out-of-sample forecasts, several authors have suggested that the “direct” forecasting method might be more robust to model misspecification. In particular, Lin and Granger (1994) compare the direct and indirect methods in the context of nonlinear models. Based on their simulation study, Lin and Granger (1994) recommend using the bootstrap predictor.[[1]](#footnote-1) Of direct interest is their finding that when the true DGP is nonlinear, forecasting with the bootstrap predictor ensures a lower mean squared predicted error (*MSPE*) than forecasting using the direct method. However, they show that when the true *DGP* is a linear autoregressive (*AR*) process, the direct method produces forecasts that are more accurate.[[2]](#footnote-2) More recently, Marcellino *et al*. (2006) provide a large empirical study regarding the forecasting performance of direct and recursive (or iterative) methods for linear models. Specifically, using several competing *AR* models they conclude that obtaining multistep ahead forecasts using the iterative method is preferable to the use of direct models. In principle, the recursive methods should produce more efficient parameter estimates. However, a direct approach to forecasting may work better when the true *DGP* is unknown in that recursive methods are be prone to bias if the one‐step‐ahead model is misspecified. Overall, Marcellino *et al*. (2006) find that iterated forecasts outperform the direct forecasts for linear models, especially for the long‐lag model specifications. Given that the literature lacks a comparable study involving nonlinear time series models, this paper aims to at least partially fill this void by comparing direct and iterated forecasts from the *LSTAR* and *ESTAR* models.

### 3. A Proposed Pretest

Teräsvirta (1994) develops a simple test to determine whether the in-sample relationship between *yt*+1 and *wt* appears to be an *LSTAR*, *ESTAR*, or linear process. The test is based on a Taylor series expansion of the function *G*(*θ*; *yt-d+h*; *c*). It is straightforward to extend his methodology to determine the relationship between *yt+h* and *wt*.

 To simplify matters, we temporarily assume *d* = *p* = 0 so that the *LSTAR* model becomes:

  (10)

 Let *z* = *θ*(*yt* − *c*) and  [1 + exp(−*z*)]−1. The approach consists in taking a third‐order Taylor series approximation of *g* with respect to *z* evaluated *z* = 0. Although the algebra is tedious, the logistic process can be approximated by a fourth-order polynomial in *yt*. After using Teräsvirta’s (1994) methodology, it possible to approximate (10) as the following fourth-order polynomial in *yt*

 *yt*+1 = *αyt* + 0.5*βyt* + 0.25*βytz* − *βytz*3/48 + *εt*+1 (11)

so that

.

 In contrast to an *LSTAR* process, an *ESTAR* series assumes a symmetric smooth adjustment between regimes. If we continue to assume *d* = *p* = 0, the ESTAR model can be written as

 *yt*+1 = *αyt* + *βyt*(1 − exp(−*θ*(*yt* − *c*)) + *εt*+1 = *αyt* + *βyt*(1 − exp(−*z*2)) + *εt*+1 (12)

where .

 Interestingly, since the ESTAR model is symmetric, the term *z*3 does not appear in the third-order Taylor series approximation of (12). Consider

 *yt*+1 = *αyt* + *βytz*2 = *αyt* + *βytθ*(*yt* – *c*)2

 The general point is that it is straightforward to conduct a Lagrange multiplier test for LSTAR and ESTAR processes. First, estimate the most appropriate autoregressive model and save the residual series {*et+*1}. Second, estimate an auxiliary regression equation in the form

 *et+*1 = *a*0 + *a*1*yt* + … + *apyt–p* + *a*11*ytyt–d* + … + *a*1*pyt–pyt–d* + *a21yt* + … + *a*2*pyt*

 + *a*31*yt* + … + *a*3*pyt*+ *vt.*

 Under the null hypothesis of linearity all of the coefficients should be equal to zero. Hence, it is possible to use a standard *F*-test to obtain the significance level of the auxiliary regression. If the null hypothesis of linearity is rejected, it is possible to go on and test for the presence of LSTAR versus ESTAR behavior. The test is possible because the Taylor series expansion of *z* in an ESTAR model does not contain the term *z*3. Hence, if it is not possible to reject the null hypothesis of ESTAR behavior (i.e., *a*31 = … = *a*3*p* = 0), conclude that there is ESTAR adjustment. A rejection of this null hypothesis suggests there is LSTAR adjustment.

*A Nonlinear Forecasting Test*

 The key to understanding our test is to recognize that the out-of-sample forecasts of LSTAR models have an LSTAR form and that the out-of-sample forecasts of ESTAR models have an ESTAR form. The intuition is clear. For an LSTAR model, the degree of autoregressive persistence is lower of one side of the attractor than the other.

 Panel a of Figure 1 shows 1-step ahead, 2-step ahead, 3-step ahead and 6-step ahead forecasts from the ESTAR model

*yt*+1 = 0.75*yt*(1 − exp[−2(*yt*)2] + *εt*+1

 Specifically, for each value of *yt* in the interval −2 to +2 using steps of 0.005, we obtained random draws of *ε*t+1 through *εt*+6 to construct the corresponding values of *yt*+1 through *yt+*6. As such, each of the 200 simulated series represent six realizations of a series *yt+h* for *h* = 1, … 6, conditional on the initial value of *yt*. We repeated this process 10,000 times and constructed the sample averages in order to obtain estimates of the conditional means *Eyt+h*|*yt*. The four lines in Panel a of Figure 1 show the 1-step ahead, 2-step ahead, 3-step ahead, and 6-step ahead conditional forecasts. The initial values for *yt* run along the horizontal axis and the forecasts run along the vertical axis. If, for example, *yt* = −1, it is possible to read off the 1-, 2-, 3- and 6-step ahead forecasts as −0.66, −0.47, −0.34 and −0.10, respectively.

 Consider the 1-step ahead forecasts indicated by the solid line in the figure. When the initial value is relatively far from zero, the slope of the line is approximately 0.75 reflecting the fact that 75 percent of the value of *yt* persists into period *t*+1. However, consistent with ESTAR adjustment, the line is flatter in the center reflecting the fact that the degree of autoregressive persistence is much smaller than 0.75 for values of *yt* that are close to zero. Notice that the same pattern holds for the constructed values of *Eyt+h*|*yt.* However, as *h* increases, all forecasts approach the unconditional mean of the series; as such, the forecast functions become flatter as the horizon, *h*, increases. The reasoning is straightforward: when *yt* is far from (close to) the attractor, the series is relatively persistent so that all values of *y+h* tend to be far from (close to) the attractor as well. As such, the functional relationship between *yt* and *yt*+1 is similar in form to the relationship between *yt* and all subsequent values of *yt+h*. Changing the magnitudes of *p* and *d,* or the centrality parameter *θ* does not alter the essential point that it is reasonable to approximate the relationship between *wt* and *Eyt+h*|*wt* as

 *Eyt+h*| *wt* = *α*(*h*)*wt* + *β*(*h*)*wt*[1 − exp(−*θ*(*h*)(*yt−d* − *c*)2] (13)

where the notation is designed to indicate that the actual parameter values of *α*, *β* and *θ* will depend on the forecast horizon *h*.

 The intuition is only slightly more difficult for an LSTAR process because the mean of an LSTAR process generally differs from the attractor. Consider

*yt*+1 = 0.75*yt*/[1 + exp(−2*yt*)] + *εt*+1

 With this particular LSTAR process, persistence is greater for positive values of *yt* than for negative values. As such, the unconditional mean of the series is approximately 0.5 even though the attractor is zero. The multi-step forecasts *Eyt+h*|*yt* shown in Panel *b* of Figure 1 were constructed in the same fashion as those for the ESTAR process. It should be clear that the 1-step ahead forecast function (shown by the solid line) has a characteristic LSTAR shape. For values of *yt* below zero, the forecasts of *Eyt*+1|*yt* are all very close to zero. For positive values of *yt*, the forecasts are similar to those from the linear model *yt*+1 = 0.75*yt* + *εt*+1. The important point is that this LSTAR shape also holds for the multi-step ahead forecasts. Although the forecast function flattens out (and simply approaches a value approximately equal to 0.5) as the forecasting horizon increases, the functions *Eyt+h*|*yt* have the same general LSTAR shape as *Eyt+*1|*yt*. Analogously to the LSTAR model, we can approximate the multi-step ahead relationship between *yt*+*h* and *wt* as relationship in the LSTAR form

 *Eyt+h*| *wt* = *α*(*h*)*wt* + *β*(*h*)*wt/*[1 + exp(−*θ*(*h*)(*yt−d* − *c*)] (14)

 In a sense, (13) and (14) are nonlinear analogues of the direct forecasting methodology used in making multi-step ahead linear forecasts. To the extent that (13) and (14) are reasonable approximations to ESTAR and LSTAR forecast functions, we can follow Teräsvirta’s (1994) methodology and use a third-order Taylor series expansion to test for the presence of nonlinearity for *h*-step ahead forecasts. To begin, we can use (14) to construct an *LM*‐type test for (non)linearity at the forecasting horizon *h*. Consider



*where* *j* = 1, 2, 3 and the null hypothesis is =0.

 If the errors are normal, *LM ~ N*(3*p*) (i.e., the degrees of freedom represent the number of variates in the nonlinear approximation). The exponential version of the test (*ESTAR*) is written in a similar manner. However, the fourth order term drops after the expansion such that the null hypothesis becomes *H*0: . In this case the test statistic is distributed as *LM ~ N*2(2*p*).

Several remarks can be made about the nature of the test. First, the *LM*‐type tests can be carried out as *F* tests. The true significance level may then be approximately close to the nominal value and, according to the discussion in pages 174 − 175 of Harvey (1990), the power may even be higher than that of the asymptotic *N*2. Moreover, the *N*2 test may suffer from size distortions when the maximum lag *p* is large relative to the sample size. Second, this testing framework does not exactly distinguish between the alternatives of an *LSTAR* or *ESTAR*, respectively. However, following Teräsvirta (1994) one may carry out a series of consecutive *F*‐tests to choose between the two. For instance, once linearity is rejected one may test the null *H*01:  against the alternative *H*11: . A rejection of this null may be interpreted as a rejection of the *ESTAR* alternative. Further, in the special case where *c* = 0, then  for an *LSTAR* process whereas for an *ESTAR* one. Thus, one may test  using another *F* test. If one fails to reject *H*02 then one may conclude the correct model is an *LSTAR.* However, a rejection is inconclusive. Finally, one can test *H*03:  against . When one rejects *H*03 then one may conclude that the model is an *LSTAR,* whereas in the opposite situation one may conclude that the model is of *ESTAR* form. Therefore, an *ESTAR* model may be chosen when accepting *H*03 after rejecting *H*02.

Of course, how well the approximations in (13) and (14) work in practice is an empirical issue. We consider some of the small sample properties of the test in the next section.

# 4. Small Sample Properties

This section performs several small‐scale Monte‐Carlo simulations to investigate the size and power performance of the linearity test at various forecasting horizons. We employ two *LSTAR* models and an *ESTAR* model that have been used in the time-series literature. The first *LSTAR* model(denoted by LSTAR1) is used in Teräsvirta (1994):

 *yt* = 1.80*yt*−1 − 1.06*yt*−2 + (0.02 − 0.9*yt*−1 + 0.795*yt*−2)[1 + exp(−*θ*(*yt*−1 − 0.02))]−1 + *εt* (15)

*where εt* is standard normally distributed. The second *LSTAR* model (denoted by *LSTAR2*) is used in Enders (2010):

 *yt* = 1 + 0.9*yt*−1 + (−3 − 1.7*yt*−1)[1 + exp(−*θ*(*yt*−1 − 5))]−1 + *εt* (16)

The following *ESTAR* model (denoted by *ESTAR1*) was estimated by Teräsvirta (1994):

 *yt* = 1.91*yt*−1 − 1.18*yt*−2 + (0.0076 − 1.07*yt*−1 + 1.18*yt*−2)[1 + exp(−*θ*(*yt*−1 + 0.086)2)]−1 + *εt* (17)

 In our simulations with the three *STAR* processes, *θ* takes the values {0, 5, 10, 20}. Each experiment uses a 5% nominal size, 5000 replications, and sample sizes of *T* = 150 and *T* = 250. Table 1 shows the results for the size simulations (i.e.,*θ* = 0) for each process up to twelve forecasting horizons. For *θ* = 0, Panels *a* through *c* of Figure 2 plot the same results. Several findings emerge. First, the test using the *LSTAR1* process is somewhat oversized while the test using *LSTAR2* has very good size properties. The degree of over‐rejection seems to increase slightly with the sample size for *LSTAR1* while for the *LSTAR2* process it seems negligible. Figure 2(c) suggests that the size for *ESTAR1* can be very poor regardless of the sample size. The reason for the poor size performance is that under the null of linearity (i.e., *θ* = 0), the characteristic roots of Teräsvirta’s (1994) model *yt* − 1.91*yt*-1 + 1.18*yt*-2 equal 0.955 ± 0.518*i* and so lie outside of the unit circle. Notice that the empirical size for all models depends on the forecast horizon. As the forecast horizon lengthens beyond 7 or 8 periods, the forecasts from the nonlinear models become more similar to those from a linear model.

Tables 2 and 3 display the results corresponding to the power simulation exercises. Before proceeding, it is important at this point to describe the differences and similarities between those two types of *STAR* models. An *ESTAR* model can be well approximated by an *LSTAR* model if most of the data generated by the model lie above the value of the threshold parameter *c*. As such, only the increasing portion of the transition function matters and this can be well mapped by a monotonically increasing function of *LSTAR* type. Conversely, an *LSTAR* model may be well approximated by an *ESTAR* model provided that the move from one regime to the other is not quick. Therefore, the simulations below describe a wide variety of competing testing frameworks to help when one does not know *a priori* how many terms to include in the Taylor series approximation.

Table 2 reveals several findings concerning the power of the test when the true *DGPs* are *LSTAR* processes. As  increases from 5 to 20, the transition function changes from an approximately linear to a threshold with two states regime. Accordingly, the power increases in for both *LSTAR* processes. Also, Panels *a* – *f* of Figure 3 show that the power of the test also increases with the sample size for both processes. Comparing directly the two *DGPs*, it seems that the *LSTAR2* process displays slightly higher power at all horizons for both samples. Finally, for both *DGPs* it is evident that the power decreases with the forecasting horizon.[[3]](#footnote-3) This follows as forecasts from nonlinear models should be very close to those from linear models as the forecast horizon gets large.

Table 3 shows the results from the power simulation exercises where the *DGP* is an *ESTAR* process. Panels *a* – *c* of Figure 4 plot the results in Table 3. Note that for *ESTAR1*, the power of the test can be quite high, even at long forecasting horizons. The power of the test decreases with *θ* but it increases with the sample size. The fact that the power of the test (for the ESTAR process) decreases as *θ* increases makes sense since as *θ* approaches infinity the model essentially becomes linear.

# 5. Forecasting Nonlinear Real Exchange Rates

We now apply our test to a set of bilateral and real effective exchange rates from several OECD countries. Extensive research has demonstrated the nonlinear nature of the exchange rates. For instance, several recent studies find evidence in favor of nonlinear but globally stationary real exchange rates with smooth adjustment (see Sollis *et al.*.(2002), Kapetanois *et al.*. (2003) and Bec *et al.* (2004)). These findings support theoretical models (e.g. Piet *et al*. (1995)) which suggest that when taking into account the effects of transaction costs, deviations from the law of one price that are nonlinear in nature may arise. These tests have considered both forms of adjustment over time, symmetric and asymmetric, that can be modeled with *ESTAR* and *LSTAR* processes, respectively.

Using an *ESTAR* process to model the exchange rates implies that the appreciation and the depreciation of real exchange rates have rather similar dynamic structures. An *ESTAR* model can thus represent an exchange rate that returns from an undervalued state towards the equilibrium rate implied by the Purchasing Power Parity (*PPP*) hypothesis in approximately the same fashion as it goes from an overvalued state towards the equilibrium level. In contrast, an *LSTAR* process assumes that the real exchange rate behavior displays asymmetric dynamics towards the equilibrium depending on the sign of shocks (i.e., positive or negative).

In order to illustrate our test we do not presuppose that the real rates are all ESTAR processes. Since the LSTAR test nests the ESTAR test, we allow our test to determine the form of the adjustment process. Although a number of papers have modelled real exchanges rates as ESTAR processes, a number of others indicate that there is no *a priori* reason to impose symmetric adjustment of the real rate to its long run equilibrium. After all, if one country’s price level is more sticky in the downward directions than is another’s, the real rate can display threshold or LSTAR behavior rather ESTAR adjustment. For example, Enders and Falk (1988), Sarantis (1999), Imbs, et al. (2003), and Shintani, Terada-Hagiwara, and Yabu (2013) find that an asymmetric adjustment model best explains the behavior of real exchange rates.

The data consists of monthly observations over the January 1975 to December 2013 period. We have included only the period of flexible exchange rates. However, for most European countries the sample ends in the last quarter of 1998 when the *Euro* has replaced the national currencies of the countries that joined the single monetary system. Table 4 reports the exact sample range for each country and rate. For completeness, we employ three different measures of each country’s real exchange rate in relation to the U.S.dollar. The first measure is the bilateral real exchange rate constructed as the natural log of the product of the number of foreign currency units per U.S. dollar and the U.S. Producer (Wholesale) Price Index (*PPI/WPI*) divided by each nation’s *WPI/PPI*. The second version constructs bilateral real exchange rates using Consumer Price Indices, also in log form. The third exchange rate measure is the natural log of the real effective exchange rate (*REER*) as constructed by the IMF.

Table 4 details the findings from applying our test to each of the real exchange rate series at one, four, eight, and twelve forecasting horizons. Note that our test at the one step ahead horizon is equivalent to Teräsvirta (1994) in-sample test. The appropriate order of integration of the real exchange rates is debatable. However, we follow traditional forecasting procedures and first difference all of the real rates in order to ensure stationarity. As shown in Table 4, a majority of the CPI-based RERs display linear adjustment at all forecasting horizons, while a majority of PPI-based RERs and REERs display some type of nonlinear adjustment at least for one of the forecasting horizons. Across all three exchange rate types, the most of the instances in which the null of linearity is not rejected occur in countries that are part of the Euro zone.[[4]](#footnote-4)

For any particular country, the results of the test can differ according to which real rate is to be forecasted and according to the forecast horizon. For example, we do not reject the null of for Austria using the PPI and CPI‐based bilateral real exchange rates (*RERs*). Austria’s real effective exchange rate (*REER*), at the one and four steps ahead forecasts display nonlinear adjustment. When the null of linearity is rejected, an LSTAR model is selected in most instances. Nevertheless, an *ESTAR* model may be most appropriate for some horizons whereas an *LSTAR* model is most appropriate for other forecasting horizons. For instance, Germany’s REER the four step ahead displays linear behavior, while the first, eighth and twelfth step display nonlinear adjustment of LSTAR type. Since forecasts from nonlinear models approach the unconditional mean of the series as the forecasting horizon increases, it is not surprising the null of linearity tends to rejected at the shorter forecasting horizons.[[5]](#footnote-5)

Thus, the linearity test appears to provide some useful insights regarding the adequacy of fitting and forecasting with a *STAR* model. Next, the following subsection compares the forecasting performance of the recursive and direct methods, for several linear and *STAR* models.

## *Direct vs. Recursive Forecasting Methods*

Given the results of our test, we select a mix of countries to use for an out of sample forecasting exercise: Australia’s *REER*, Austria’s *PPI* based *RER*, Austria’s REER, Canada’s PPI-based RER, France’s *PPI*‐based *RER*, Germany’s *PPI*‐based *RER*, Norway’s *REER*, Norway’s PPI-based RER, and Spain’s *REER*. All forecasts are performed for the four, eight, and twelve forecast horizons. Notice that the selection includes rates for which the null of linearity cannot be rejected at the 5% level at any forecast horizon (i.e., Australia’s REER, Austria’s PPI-based RER), rates for which the null is only mildly rejected at a single forecast horizon (i.e., Canada’s and France’s PPI-based RER), and rates for which the null is strongly rejected at more than one forecast horizon (i.e., Germany’s and Norway’s PPI-based RER, and Austria’s, Norway’s and Spain’s REER).

 We first examine whether the results of our tests are ignored and report the results then the nine series are forecasted using a linear model. Following Marcellino *et al*. (2006), Table 5 reports the results from four competing linear models: an *AR* with a fixed lag of four (i.e., AR(4)), an *AR* with twelve fixed lags (i.e., AR(12)), and two *ARs* estimated using *BIC* and *AIC* criteria to determine the optimum lag length (i.e., denoted AR(BIC) and AR(AIC), respectively). The forecasting performance is evaluated using a set of two criteria: the root-mean squared error (*RMSE*) and the mean absolute percentage error (*MAPE*). The forecasts are computed rolling over the entire sample as follows. We start with a sample of 133 observations for each series and compute the out-of-sample forecasts at the 4, 8, and 12 step horizons. Then, we add one more observation recursively and keep increasing the size of the estimation sample until we reach the end of the sample for each series. At each step, we gather the out-of-sample forecasts corresponding to each forecast horizon and compute the forecasts errors. We average the results and obtain the two statistics (i.e., RMSE and MAPE) by making sure each series contains the same number of observations.[[6]](#footnote-6)

The results, reported in Table 5, yield several findings. Not surprisingly, the *RMSEs* appear to increase with the forecasting horizon. Also as anticipated, the best forecasting models tend to be those with the fewest lags: those from an AR(4) or an AR (BIC). More importantly, for the series for which our linearity test strongly rejects the null, the direct method strongly dominates the recursive (or iterative) method (e.g., Germany and Norway’s PPI-based RER). For instance, at the twelfth forecast horizon the RMSE of the direct method is 11 and 73 times lower than that of the iterative one, respectively when comparing the best forecasting models using each method. However, for the series for which the null of linearity is either not rejected or just mildly rejected, the difference between the direct and iterative methods is not as severe. For example, at the twelfth forecast horizon for those series the direct method’s RMSE is roughly 3 to 4 times lower. Thus, when the modeling process appears strongly misspecified (i.e., using a linear forecasting method while the DGP appears nonlinear) the direct method is clearly preferred. These findings highlight the usefulness of pre‐testing for linearity at various forecasting horizons.

Next, as reported in Table 6, we obtain nonlinear forecasts for each series at each of the forecasting horizons. Since the LSTAR model nests the ESTAR model, we do not impose the ESTAR restriction that the third partial derivative of our expansion is zero and simply estimate each series an LSTAR process. As discussed in Levich and Poti (2014), the in-sample properties of a model may be quite different from its out-of-sample properties. As in Thomakos and Guerard (2004), we seek to determine the forecasting performance of the models and not to uncover the precise data generating process of the real exchange rate series.

We obtain the nonlinear forecasts using five different methods. In the first case, (Nonsimulation I) we fit an LSTAR to the sample by forcing the transition variable to be the first lag of the differenced rate and the centrality parameter *c* to be the sample mean. In the second case (Nonsimulation II), we find the delay parameter and centrality parameters using the values that minimize the Schwartz Bayesian criterion of the in-sample fit. Third, we obtain the forecasts using a Monte Carlo approach drawing the shocks from a standard normal distribution fixing the centrality parameter at the mean. Fourth, we fixed the value of *c* and bootstrapped the forecasts drawing the shocks from the residuals resulting from the estimation step. Fifth, we compute the direct forecasts for the nonlinear case employing the process called Direct. We then estimate the LSTAR assuming the attractor is at lags 4, 8, and 12, respectively and thus establish the relationship between periods *t* and *t+*4, *t+*8, and *t+*12, respectively. As indicated earlier, we do not impose the zero restriction on the third derivative implied by an ESTAR model. Several conclusions emerge.

First, when comparing the nonlinear methods amongst themselves it appears that the non-simulation II, the bootstrap and the direct methods are comparable to each other. However, it appears that the bootstrap yields the lowest mean absolute percentage error in most of the cases. In turn, the static methods (i.e., the non-simulation II and direct methods, respectively) yield the lowest RMSE. Second, the iterated methods (i.e., the non-simulation II and the bootstrap) outperform the direct method when the null of linearity is strongly rejected as in the case of Germany’s PPI-based RER. As such, if our test rejects the null of linearity, it makes sense to forecast the series using the either of these two methods. Third, it appears that when comparing the Monte Carlo and the bootstrap approaches, the latter provides a lower RMSE and MAPE especially when the process appears highly nonlinear. However, when our test indicates that the series displays linear adjustment, the Monte Carlo approach generates RMSE and MAPE that are lower. This evidence confirms the Lin and Granger (1994) result that the bootstrap predictor is recommended when the DGP appears highly nonlinear.

Finally, when comparing the best forecasts from Tables 5 and 6, it appears that whenever our proposed test strongly rejects the null of linearity, the best *LSTAR* forecast yields a lower mean absolute percentage error than that of the linear counterpart(France, Germany, and Norway’s PPI-based RER). Moreover, when our test indicates that the process is linear (as in the case of Australia, Austria and Canada) the forecasts from a generally linear model (in our case using the direct method) produce better forecasts than those from a nonlinear model.

## *A Robustness Check*

Mincer and Zarnowitz (1969) and Montgomery et al. (1998) discuss the importance of benchmarking out of sample forecasts. Towards this end, Table 7 compares the naïve forecasts of a simple random walk model to the best forecasts from the linear and nonlinear exchange rate models.[[7]](#footnote-7) As Thomakos and Guerard (2004) point out, the use of a no-change model as a benchmark does not necessarily imply the existence of an underlying unit root process as such forecasts are made without reference to any specific data generating process.

 The entries in Table 7 represent the ratio of the RMSE and MAPE of the best of the linear and nonlinear methods, respectively to the RMSE and MAPE of the random walk. Again, we begin with the first 133 observations for each series and compute the out-of-sample forecasts at the 4, 8, and 12 step horizons. Then, we add one more observation recursively and keep increasing the size of the estimation sample until we reach the end of the sample for each series. At each step, we gather the out-of-sample forecasts corresponding to each forecast horizon and compute the forecasts errors. We average the results and obtain the two statistics (i.e., RMSE and MAPE) by making sure each series contains the same number of observations. The important point is that across all series and forecast horizons considered, both the linear and nonlinear models clearly dominate the forecasting performance resulting from a random walk. Thus, both the RMSE and MAPE of each of the best linear and nonlinear models are in turn lower than those of a random walk, respectively.

# 6. Conclusion

As indicated by White (2006), multi-step ahead forecasting with nonlinear models entail “potentially serious practical challenges.” Not only are such forecasts computationally difficult, but as Teräsvirta (2006) shows, there are alternative choices that can lead a researcher astray when using a nonlinear model to make multi-step ahead forecasts. To circumvent some of these problems, we propose a simple pretest to determine whether it is worthwhile to forecast using a linear or a nonlinear model. The advantage of our pretest is that one might be able to avoid estimating and fitting a nonlinear model when the relationship between the current and future values of a series is sufficiently linear. The (non)linearity test of this paper extends that of Teräsvirta (1994) in order to consider nonlinearity at various forecasting horizons. Depending on the order of the expansion, one can capture both forms of a *STAR* process (ESTAR or LSTAR). The size and power of the test are quite reasonable.

The empirical section considers several types of real exchange rates from a sample of OECD countries. We assess the properties of our test using out-of-sample forecasts at one, four, eight, and twelve forecasting steps, respectively. We use both the random walk model and linear autoregressive models as benchmarks to evaluate our test. Among the main findings, we show that when the proposed test strongly rejects the null of linearity, (*i*) a direct method of forecasting outperforms a linear iterated one, (*ii*) the bootstrap predictor of a nonlinear model (ESTAR or LSTAR) outperforms the direct method, and (*iii*) the best nonlinear model outperforms the best linear model in terms of a lower mean absolute percentage error. However, when the null is either not rejected or only mildly so, the best linear method yields forecasts that are slightly superior to those generated from a nonlinear model. These results confirm prior evidence and provide support for our proposed methodology. Finally, a set of robustness checks confirms that forecasts from a nonlinear (or linear) model display both lower RMSE and MAPE than those of a random walk model.

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**Table 1: Size simulations**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|   | **LSTAR1** | **LSTAR1** | **LSTAR2** | **LSTAR2** | **ESTAR1** | **ESTAR1** |
| Horizon */N* | 150 | 250 | 150 | 250 | 150 | 250 |
| 1 | 0.0776 | 0.0972 | 0.0450 | 0.0474 | 0.0586 | 0.1288 |
| 2 | 0.0936 | 0.1172 | 0.0448 | 0.0500 | 0.1312 | 0.1606 |
| 3 | 0.0996 | 0.1272 | 0.0462 | 0.0440 | 0.1630 | 0.1832 |
| 4 | 0.1060 | 0.1262 | 0.0436 | 0.0470 | 0.1890 | 0.2218 |
| 5 | 0.1046 | 0.1176 | 0.0388 | 0.0434 | 0.2328 | 0.2388 |
| 6 | 0.1038 | 0.1126 | 0.0446 | 0.0546 | 0.2536 | 0.2074 |
| 7 | 0.0976 | 0.1068 | 0.0426 | 0.0414 | 0.2130 | 0.1338 |
| 8 | 0.0914 | 0.1030 | 0.0456 | 0.0454 | 0.1446 | 0.1054 |
| 9 | 0.0904 | 0.0970 | 0.0422 | 0.0464 | 0.1128 | 0.1058 |
| 10 | 0.0800 | 0.0894 | 0.0516 | 0.0450 | 0.1128 | 0.1474 |
| 11 | 0.0888 | 0.0932 | 0.0426 | 0.0452 | 0.1472 | 0.1758 |
| 12 | 0.0894 | 0.1020 | 0.0520 | 0.0476 | 0.1848 | 0.0668 |

**Table 2: LSTAR Power Simulations**

|  |  |  |  |
| --- | --- | --- | --- |
|  | *θ* = 5 | *θ* = 10 | *θ* = 20 |
|  | LSTAR1 | LSTAR2 | LSTAR1 | LSTAR2 | LSTAR1 | LSTAR2 |
| Horizon */N* | 150 | 250 | 150 | 250 | 150 | 250 | 150 | 250 | 150 | 250 | 150 | 250 |
| 1 | 0.9982 | 1 | 1 | 1 | 0.998 | 1 | 1 | 1 | 0.9968 | 1 | 1 | 1 |
| 2 | 0.8854 | 0.9888 | 0.9944 | 1 | 0.8942 | 0.991 | 1 | 1 | 0.8786 | 0.9908 | 1 | 1 |
| 3 | 0.3440 | 0.5244 | 0.6826 | 0.9438 | 0.3608 | 0.5390 | 0.8700 | 0.9936 | 0.3512 | 0.5530 | 0.9282 | 0.9980 |
| 4 | 0.2312 | 0.3426 | 0.2188 | 0.4262 | 0.2504 | 0.3600 | 0.3944 | 0.6726 | 0.2442 | 0.3808 | 0.5084 | 0.7854 |
| 5 | 0.2888 | 0.4578 | 0.0896 | 0.1432 | 0.3024 | 0.4754 | 0.1866 | 0.3094 | 0.2962 | 0.4880 | 0.2632 | 0.4668 |
| 6 | 0.2504 | 0.4028 | 0.0856 | 0.1248 | 0.2760 | 0.3982 | 0.1970 | 0.3310 | 0.2536 | 0.4010 | 0.3122 | 0.5216 |
| 7 | 0.1658 | 0.2460 | 0.1222 | 0.1660 | 0.1748 | 0.2532 | 0.2638 | 0.4116 | 0.1726 | 0.2412 | 0.3970 | 0.6160 |
| 8 | 0.1276 | 0.1652 | 0.1388 | 0.1728 | 0.1320 | 0.1728 | 0.2734 | 0.3988 | 0.1314 | 0.1694 | 0.3804 | 0.5874 |
| 9 | 0.1148 | 0.1426 | 0.1372 | 0.1580 | 0.1220 | 0.1516 | 0.2446 | 0.317 | 0.1194 | 0.1492 | 0.3238 | 0.4622 |
| 10 | 0.1108 | 0.1358 | 0.1270 | 0.1402 | 0.1170 | 0.1414 | 0.2102 | 0.2358 | 0.1188 | 0.1482 | 0.2518 | 0.3174 |
| 11 | 0.1060 | 0.1400 | 0.1210 | 0.1250 | 0.1152 | 0.1434 | 0.1788 | 0.1918 | 0.1238 | 0.1494 | 0.2176 | 0.2404 |
| 12 | 0.1060 | 0.1372 | 0.1220 | 0.1264 | 0.1160 | 0.1454 | 0.1728 | 0.1882 | 0.1194 | 0.1520 | 0.2026 | 0.2244 |

**Table 3: ESTAR Power Simulations**

|  |  |  |  |
| --- | --- | --- | --- |
|  | *θ* = 5 | *θ* = 10 | *θ* = 20 |
| Horizon */N* | 150 | 250 | 150 | 250 | 150 | 250 |
| 1 | 0.9992 | 1 | 0.9976 | 1 | 0.9892 | 0.9996 |
| 2 | 0.9958 | 1 | 0.9878 | 0.9998 | 0.9414 | 0.9960 |
| 3 | 0.9208 | 0.9924 | 0.8582 | 0.9790 | 0.7610 | 0.9424 |
| 4 | 0.7108 | 0.8866 | 0.6514 | 0.8416 | 0.5836 | 0.7976 |
| 5 | 0.5076 | 0.6778 | 0.4700 | 0.6424 | 0.4212 | 0.6128 |
| 6 | 0.3624 | 0.5304 | 0.3248 | 0.4544 | 0.2892 | 0.4286 |
| 7 | 0.3166 | 0.5016 | 0.2470 | 0.3340 | 0.2166 | 0.2870 |
| 8 | 0.3238 | 0.5018 | 0.2314 | 0.2946 | 0.1886 | 0.2194 |
| 9 | 0.3132 | 0.4562 | 0.2256 | 0.2800 | 0.1858 | 0.2046 |
| 10 | 0.2758 | 0.3826 | 0.2092 | 0.2644 | 0.1780 | 0.1942 |
| 11 | 0.2182 | 0.2852 | 0.1896 | 0.2278 | 0.1702 | 0.1854 |
| 12 | 0.1684 | 0.2074 | 0.1690 | 0.1942 | 0.1592 | 0.1796 |

**Table 4: Nonlinearity Tests for Bilateral and Real Effective Exchange Rates**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| RER(PPI/WPI) |  | RER(CPI) |  | REER |  |
| Steps ahead | One | Four | Eight | Twelve | One | Four | Eight | Twelve | One | Four | Eight | Twelve |
| **Australia** |  |  | 1979:12 – 2003:6 |
|   |  |  |  |  |  |  |  |  | 1.85\*§ | 0.85 | 1.77 | 0.95 |
| **Austria** | 1975:1 – 1997:06 | 1975:1 – 1998:12 | 1975:1 - 2013:12 |
|   | 0.63 | 0.71 | 0.16 | 0.18 | 0.38 | 0.42 | 0.56 | 0.77 | 6.28\*\*\*§ | 0.02 | 3.41\*\*§ | 0.16 |
| **Canada** | 1975:1 - 2013:12 | 1975:1 - 2013:12 | 1975:2 - 2013:12 |
|   | 2.73\*\*§ | 0.75 | 0.28 | 2.49\*† | 4.12\*\*\*§ | 0.64 | 0.79 | 5.77\*\*\*§ | 2.06 | 0.44 | 1.09 | 0.09 |
| **Denmark** | 1975:1 - 2013:10 | 1975:1 - 2013:12 | 1975:1 - 2013:12 |
|   | 2.23\*§ | 1.08 | 0.34 | 0.43 | 2.23\*§ | 0.78 | 0.58 | 0.32 | 2.47\*† | 0.11 | 1.23 | 0.92 |
| **Finland** | 1975:1 – 1998:12 | 1975:1 – 1998:12 | 1975:1 - 2013:12 |
|   | 0.88 | 2.51\*† | 0.40 | 0.35 | 0.44 | 2.60\*† | 0.35 | 0.39 | -1.73\*† | 2.63\*\*§ | 2.07 | 0.58 |
| **France** | 1975:1 – 1998:12 | 1975:1 – 1998:12 | 1979:12 - 2013:12 |
|   | 1.23 | 0.80 | 0.43 | 1.81\*§ | 0.53 | 0.48 | 0.23 | 0.56 | 6.56\*\*\*§ | 0.56 | 0.58 | 0.59 |
| **Germany** | 1975:1 – 1990:12 | 1991:01 – 1998:12 | 1975:1 - 2013:12 |
|   | -2.14\*\*\*§ | 4.15\*\*\*§ | 3.61\*\*§ | 4.65\*\*§ | 0.21 | 0.73 | 3.63\*\*§ | 1.63 | 6.87\*\*\*§ | 0.57 | 4.40\*\*\*§ | 2.55\*§ |
| **Greece** | 1975:1 – 1990:12 | 1975:1 - 2013:12 | 1975:1 - 2013:12 |
|   | 0.47 | 0.64 | 0.26 | 0.42 | 0.56 | 0.70 | 0.69 | 0.61 | 3.69\*\*§ | 0.15 | -1.83\*§ | -2.34\*\*§ |
| **Ireland** | 1975:1 – 1998:12 | 1975:1 - 1998:12 | 1975:1 - 2013:12 |
|   | 4.21\*\*\*§ | 0.36 | 0.05 | 0.21 | 0.70 | 0.17 | 1.60 | 2.34\*\*§ | 4.49\*\*\*§ | 0.93 | 1.26 | 1.31 |
| **Italy** | 1975:1 – 1998:12 | 1975:1 – 1998:12 | 1980:1 - 2013:12 |
|   | 3.33\*\*§ | 0.18 | 1.73\*§ | 1.62 | 2.14\*§ | 0.97 | 0.24 | 0.66 | 11.55\*\*\*§ | 1.31 | 2.99\*\*§ | 1.53 |
| **Japan** | 1981:1 - 2013:12 | 1975:1 - 2013:12 | 1975:1 - 2013:12 |
|   | 0.76 | 0.82 | 1.47 | 0.29 | 0.25 | 0.10 | 1.67\*§ | 0.96 | 2.45\*\*§ | 4.96\*\*\*§ | 0.66 | 0.65 |
| **Luxembourg** |  | 1975:1 - 1998:12 | 1975:1 - 2013:12 |
|   |  |  |  |  | 1.03 | 1.26 | 0.60 | 0.26 | 0.98 | 0.77 | 1.62 | 0.21 |
| **Netherlands** | 1975:1 - 1998:12 | 1975:1 - 1998:12 | 1975:1 - 2013:12 |
|   | 1.11\*§ | 0.79 | 1.33 | 0.23 | 1.03 | 1.26 | 0.60 | 0.26 | 0.99 | 0.77 | 1.62 | 0.21 |
| **Norway** | 1977:1 - 2013:12 | 1975:1 - 2013:12 | 1975:1 - 2013:12 |
|   | 2.67\*\*§ | 1.99\*\*§ | 2.03 | 2.40\*† | 2.66\*\*§ | 0.93 | 0.89 | 0.76 | 1.77 | 0.55 | 2.70\*\*§ | 2.07\*\*§ |
| **New Zealand** | 1975:1 - 2013:11 |  | 1975:1 - 2013:12 |
|   | 2.22\*§ | 1.84\*§ | 0.39 | 3.55\*\*† |  |  |  |  | 1.39 | 0.91 | 0.86 | 3.36\*§ |
| **Spain** | 1975:1 - 1998:12 | 1975:1 – 1998:12 | 1980:1 - 2013:12 |
|   | 2.23\*\*§ | 0.29 | 0.21 | 0.57 | 1.84 | 0.21 | 0.20 | 1.16 | 2.58\*§ | 0.07 | 6.12\*\*\*§ | 1.91\*§ |
| **Sweden** | 1975:1 - 2013:12 | 1975:1 - 2013:12 | 1975:1 - 2013:12 |
|   | 2.08 | 0.37 | 1.12 | 0.46 | 1.76 | 0.18 | 1.09 | 0.04 | 4.19\*\*\*§ | 0.26 | 0.08 | 2.35\*§ |
| **Switzerland** | 1975:1 - 2013:12 | 1975:1 - 2013:12 | 1975:1 - 2013:12 |
|   | 1.01 | 0.76 | 2.69\*\*§ | 0.19 | 0.38 | 0.16 | 3.05\*\*§ | 0.57 | 8.80\*\*\*§ | -1.98\*§ | 0.25 | 0.87 |
| **UK** | 1975:1 - 2013:12 | 1975:1 - 2013:12 | 1975:1 - 2013:12 |
|   | 1.64\*§ | 1.84\* | 0.12 | 0.71 | 2.70\*§ | 1.88\*† | 0.29 | 0.79 | 1.81\*§ | 0.66 | 0.42 | 7.87\*\*\*§ |

\* Significance at the 10% level; \*\* Significance at the 5% level; \*\*\* Significance at the 1% level; † The ESTAR model is selected over the LSTAR model; when the † symbol is missing then the LSTAR model is selected; § The null hypothesis is rejected suggesting that the LSTAR model is preferred. Under each country we report the full sample available.

As discussed in the text, the *F*-values reported above were carried out sequentially. Specifically, we for test for the null of linearity and then used the sequential tests for LSTAR versus ESTAR behavior. In some instances the entry in the table is the *t*-test for the null hypothesis *β*1 = 0.

**Table 5: Four Methods of Creating Linear Forecasts**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Country  |  | **AR(4)** | **AR(12)** | **AR(BIC)** | **AR(AIC)** |
| **AUS REER** |  | *4 steps*  | *8 steps*  | *12 steps*  | *4 steps*  | *8 steps*  | *12 steps*  | *4 steps*  | *8 steps*  | *12 steps*  | *4 steps*  | *8 steps*  | *12 steps*  |
| Iterated | RMSE | 3.993 | 5.382 | 6.701 | 4.275 | 5.724 | 7.137 | 4.047 | 5.499 | 6.766 | 4.275 | 5.724 | 7.137 |
| Iterated | MAPE | 0.039 | 0.052 | 0.066 | 0.041 | 0.057 | 0.072 | 0.038 | 0.053 | 0.532 | 0.041 | 0.057 | 0.594 |
| Direct | RMSE | 1.940 | 1.884 | 1.909 | 2.050 | 2.068 | 2.093 | 1.961 | 1.954 | 1.991 | 1.961 | 1.954 | 1.991 |
| Direct | MAPE | 0.019 | 0.019 | 0.019 | 0.020 | 0.020 | 0.021 | 0.019 | 0.020 | 0.019 | 0.019 | 0.020 | 3.965 |
| **AUT PPI RER** |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 0.079 | 0.116 | 0.158 | 0.084 | 0.130 | 0.506 | 0.078 | 0.115 | 0.495 | 0.088 | 0.139 | 0.513 |
| Iterated | MAPE | 0.026 | 0.039 | 0.055 | 0.027 | 0.045 | 0.082 | 0.025 | 0.038 | 0.099 | 0.028 | 0.047 | 0.106 |
| Direct | RMSE | 0.031 | 0.030 | 0.030 | 0.033 | 0.033 | 0.033 | 0.031 | 0.032 | 0.032 | 0.031 | 0.032 | 0.032 |
| Direct | MAPE | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.000 | 0.010 | 0.010 | 0.001 |
| **AUT REER\***§ |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 1.393 | 2.027 | 2.661 | 1.463 | 2.160 | 2.958 | 1.380 | 2.013 | 91.327 | 1.464 | 2.161 | 91.337 |
| Iterated | MAPE | 0.011 | 0.017 | 0.022 | 0.012 | 0.018 | 0.024 | 0.011 | 0.016 | 82.892 | 0.012 | 0.018 | 82.911 |
| Direct | RMSE | 0.592 | 0.593 | 0.591 | 0.612 | 0.617 | 0.619 | 0.598 | 0.600 | 0.602 | 0.598 | 0.600 | 0.602 |
| Direct | MAPE | 0.005 | 0.005 | 0.005 | 0.005 | 0.005 | 0.005 | 0.005 | 0.005 | 0.004 | 0.005 | 0.005 | 0.362 |
| **CAN PPI RER\***§ |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 0.068 | 0.101 | 0.133 | 0.072 | 0.111 | 0.152 | 0.067 | 0.100 | 0.131 | 0.072 | 0.111 | 0.152 |
| Iterated | MAPE | 0.034 | 0.051 | 0.071 | 0.035 | 0.058 | 0.080 | 0.033 | 0.050 | 0.011 | 0.035 | 0.058 | 0.014 |
| Direct | RMSE | 0.027 | 0.026 | 0.027 | 0.029 | 0.028 | 0.028 | 0.027 | 0.027 | 0.027 | 0.027 | 0.027 | 0.027 |
| Direct | MAPE | 0.013 | 0.013 | 0.013 | 0.014 | 0.014 | 0.014 | 0.013 | 0.013 | 0.000 | 0.013 | 0.013 | 0.001 |
| **France PPI RER\***§ |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 0.426 | 0.308 | 0.129 | 0.427 | 0.311 | 0.148 | 0.426 | 0.308 | 0.127 | 0.427 | 0.311 | 0.148 |
| Iterated | MAPE | 0.093 | 0.079 | 0.068 | 0.094 | 0.085 | 0.077 | 0.093 | 0.078 | 0.010 | 0.094 | 0.085 | 0.014 |
| Direct | RMSE | 0.422 | 0.293 | 0.026 | 0.422 | 0.294 | 0.028 | 0.422 | 0.293 | 0.027 | 0.422 | 0.293 | 0.027 |
| Direct | MAPE | 0.074 | 0.043 | 0.013 | 0.075 | 0.044 | 0.013 | 0.074 | 0.043 | 0.000 | 0.074 | 0.043 | 0.001 |
| **Germany PPI RER\*\*\***§ |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 0.016 | 0.026 | 0.062 | 0.020 | 0.031 | 0.084 | 0.018 | 0.027 | 0.084 | 0.020 | 0.031 | 0.087 |
| Iterated | MAPE | 0.003 | 0.004 | 0.007 | 0.003 | 0.005 | 0.009 | 0.003 | 0.004 | 0.002 | 0.003 | 0.005 | 0.002 |
| Direct | RMSE | 0.006 | 0.006 | 0.006 | 0.011 | 0.013 | 0.016 | 0.006 | 0.006 | 0.006 | 0.006 | 0.006 | 0.006 |
| Direct | MAPE | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.001 | 0.001 | 0.000 | 0.001 | 0.001 | 0.000 |
| **Norway REER\*\***§ |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 2.854 | 3.858 | 4.734 | 2.935 | 3.968 | 4.836 | 2.865 | 3.867 | 4.758 | 2.935 | 3.968 | 4.761 |
| Iterated | MAPE | 0.021 | 0.029 | 0.036 | 0.022 | 0.030 | 0.037 | 0.021 | 0.030 | 0.229 | 0.022 | 0.030 | 0.228 |
| Direct | RMSE | 1.259 | 1.272 | 1.251 | 1.276 | 1.292 | 1.359 | 1.292 | 1.277 | 1.280 | 1.292 | 1.277 | 1.280 |
| Direct | MAPE | 0.009 | 0.009 | 0.009 | 0.009 | 0.010 | 0.010 | 0.009 | 0.009 | 0.017 | 0.009 | 0.009 | 1.637 |
| **Norway PPI RER\*\***§ |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 0.071 | 0.112 | 0.155 | 0.084 | 0.116 | 0.157 | 0.076 | 0.105 | 0.140 | 0.083 | 0.112 | 0.148 |
| Iterated | MAPE | 0.030 | 0.046 | 0.059 | 0.034 | 0.048 | 0.062 | 0.031 | 0.045 | 0.011 | 0.033 | 0.047 | 0.012 |
| Direct | RMSE | 0.031 | 0.031 | 0.031 | 0.035 | 0.033 | 0.033 | 0.032 | 0.032 | 0.002 | 0.032 | 0.032 | 0.032 |
| Direct | MAPE | 0.013 | 0.013 | 0.013 | 0.014 | 0.014 | 0.014 | 0.013 | 0.013 | 0.000 | 0.013 | 0.013 | 0.001 |
| **Spain REER\*\***§ |  | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| Iterated | RMSE | 2.146 | 3.303 | 4.771 | 3.035 | 4.737 | 5.787 | 2.023 | 3.112 | 4.614 | 3.035 | 4.737 | 5.787 |
| Iterated | MAPE | 0.016 | 0.027 | 0.037 | 0.018 | 0.030 | 0.040 | 0.015 | 0.025 | 0.225 | 0.018 | 0.030 | 0.356 |
| Direct | RMSE | 0.801 | 0.872 | 0.825 | 1.101 | 1.029 | 0.877 | 0.874 | 0.855 | 0.837 | 0.874 | 0.855 | 0.837 |
| Direct | MAPE | 0.006 | 0.006 | 0.006 | 0.007 | 0.007 | 0.006 | 0.006 | 0.006 | 0.007 | 0.006 | 0.006 | 0.701 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Mean ‐ mean of forecasts; MAPE ‐ mean absolute percentage error. In this table, **\*** means that we reject the null of linearity in favor of the alternative of an LSTAR model for either one of 4, 8, or 12 steps ahead; **\*\*** means that we reject the null of linearity in favor of the alternative of an LSTAR model for two of 4, 8, or 12 steps ahead; **\*\*\*** means that we reject the null of linearity in favor of the alternative of an LSTAR model for all of 4, 8, or 12 steps ahead.

|  |
| --- |
| **Table 6: Five Methods of Creating Nonlinear Forecasts** |
| Country |  | **Nonsimulation I** | **Nonsimulation II** | **Monte Carlo** | **Bootstrap** | **Direct** |
|  | 4 step | 8 step | 12 step | 4 step | 8 step | 12 step | 4 step | 8 step | 12 step | 4 step | 8 step | 12 step | 4 step | 8 step | 12 step |
| **AUS REER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 3.874 | 5.082 | 6.084 | 3.863 | 5.168 | 5.910 | 3.873 | 5.110 | 6.210 | 3.923 | 5.176 | 6.134 | 84.206 | 84.411 | 84.533 |
|  | MAPE | 0.036 | 0.051 | 0.060 | 0.037 | 0.052 | 0.429 | 0.036 | 0.051 | 0.061 | 0.183 | 0.318 | 0.442 | 0.645 | 0.649 | 0.653 |
| **AUT PPI RER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 0.070 | 0.089 | 0.101 | 0.064 | 0.078 | 0.087 | 0.070 | 0.089 | 0.101 | 0.073 | 0.104 | 0.135 | 2.287 | 2.294 | 2.299 |
|  | MAPE | 0.024 | 0.032 | 0.034 | 0.022 | 0.028 | 0.022 | 0.024 | 0.032 | 0.034 | 0.002 | 0.004 | 0.007 | 0.548 | 0.552 | 0.557 |
| **AUT REER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 1.446 | 2.160 | 2.856 | 1.395 | 1.965 | 2.502 | 1.333 | 1.940 | 2.505 | 1.336 | 1.939 | 2.498 | 51.752 | 51.735 | 51.736 |
|  | MAPE | 0.011 | 0.017 | 0.023 | 0.011 | 0.015 | 0.062 | 0.010 | 0.015 | 0.020 | 0.018 | 0.037 | 0.062 | 0.317 | 0.321 | 0.322 |
| **CAN PPI RER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 0.039 | 0.058 | 0.072 | 0.038 | 0.055 | 0.064 | 0.039 | 0.058 | 0.072 | 0.039 | 0.057 | 0.069 | 0.041 | 0.057 | 0.073 |
|  | MAPE | 0.905 | 1.197 | 1.443 | 0.364 | 0.220 | 0.022 | 0.905 | 1.300 | 1.560 | 0.022 | 0.045 | 0.077 | 1.397 | 1.670 | 5.234 |
| **France CPI RER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 0.062 | 0.086 | 0.110 | 0.059 | 0.078 | 0.089 | 0.062 | 0.086 | 0.108 | 0.062 | 0.086 | 0.110 | 0.057 | 0.062 | 0.077 |
|  | MAPE | 0.031 | 0.047 | 0.060 | 0.030 | 0.042 | 0.005 | 0.031 | 0.047 | 0.059 | 0.002 | 0.005 | 0.008 | 0.028 | 0.034 | 0.039 |
| **Germany PPI RER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 0.019 | 0.032 | 0.048 | 0.013 | 0.017 | 0.021 | 0.019 | 0.032 | 0.048 | 0.015 | 0.022 | 0.031 | 0.017 | 0.020 | 0.026 |
|  | MAPE | 0.003 | 0.005 | 0.007 | 0.002 | 0.003 | 0.000 | 0.003 | 0.005 | 0.007 | 0.000 | 0.000 | 0.000 | 0.003 | 0.004 | 0.005 |
| **Norway REER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 2.750 | 3.611 | 4.266 | 2.720 | 3.571 | 4.179 | 2.717 | 3.520 | 4.139 | 2.725 | 3.547 | 4.150 | 2.518 | 2.898 | 3.065 |
|  | MAPE | 0.021 | 0.027 | 0.033 | 0.020 | 0.027 | 0.177 | 0.020 | 0.027 | 0.032 | 0.076 | 0.128 | 0.174 | 0.020 | 0.023 | 0.024 |
| **Norway PPI RER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 0.072 | 0.104 | 0.129 | 0.070 | 0.099 | 0.117 | 0.072 | 0.104 | 0.128 | 0.071 | 0.102 | 0.124 | 0.069 | 0.091 | 0.111 |
|  | MAPE | 0.031 | 0.047 | 0.058 | 0.030 | 0.043 | 0.008 | 0.031 | 0.047 | 0.058 | 0.003 | 0.006 | 0.009 | 0.030 | 0.040 | 0.050 |
| **Spain REER** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | RMSE | 2.155 | 3.232 | 4.396 | 1.790 | 2.716 | 3.762 | 2.142 | 3.214 | 4.347 | 2.187 | 3.325 | 4.537 | 2.100 | 3.075 | 3.868 |
|  | MAPE | 0.014 | 0.022 | 0.029 | 0.013 | 0.021 | 0.151 | 0.014 | 0.022 | 0.029 | 0.050 | 0.115 | 0.217 | 0.015 | 0.022 | 0.029 |

**Table 7: A Random Walk versus the best of linear and nonlinear models**

|  |  |  |  |
| --- | --- | --- | --- |
| Country  | **Random Walk** | **Best of Nonlinear Methods** | **Best of Linear Methods** |
| **AUS\_REER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.923 | 0.832 | 0.712 | 0.463 | 0.308 | 0.230 |
| **MAPE** | 1 | 1 | 1 | 0.922 | 0.863 | 0.784 | 0.490 | 0.323 | 0.246 |
| **AUT\_PPI\_RER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.813 | 0.669 | 0.518 | 0.392 | 0.260 | 0.181 |
| **MAPE** | 1 | 1 | 1 | 0.083 | 0.112 | 0.133 | 0.382 | 0.254 | 0.007 |
| **AUT REER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.979 | 0.966 | 0.950 | 0.434 | 0.295 | 0.224 |
| **MAPE** | 1 | 1 | 1 | 0.945 | 0.917 | 0.918 | 0.412 | 0.276 | 0.167 |
| **CAN PPI RER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.987 | 0.973 | 0.922 | 0.695 | 0.469 | 0.383 |
| **MAPE** | 1 | 1 | 1 | 0.023 | 0.024 | 0.009 | 0.013 | 0.007 | 0.0002 |
| **France CPI RER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.913 | 0.684 | 0.610 | 6.706 | 3.217 | 0.211 |
| **MAPE** | 1 | 1 | 1 | 0.002 | 0.004 | 0.005 | 0.074 | 0.043 | 0.0004 |
| **Germany PPI RER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.729 | 0.435 | 0.2644 | 0.327 | 0.147 | 0.071 |
| **MAPE** | 1 | 1 | 1 | 0.014 | 0.014 | 0.006 | 0.269 | 0.121 | 0.0005 |
| **Norway REER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.981 | 0.901 | 0.904 | 0.439 | 0.311 | 0.015 |
| **MAPE** | 1 | 1 | 1 | 0.096 | 0.139 | 0.141 | 0.422 | 0.297 | 0.234 |
| **Norway PPI RER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.981 | 0.901 | 0.904 | 0.439 | 0.311 | 0.015 |
| **MAPE** | 1 | 1 | 1 | 0.096 | 0.139 | 0.141 | 0.434 | 0.302 | 0.0003 |
| **Spain REER** | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* | *4 steps* | *8 steps* | *12 steps* |
| **RMSE** | 1 | 1 | 1 | 0.819 | 0.807 | 0.793 | 0.367 | 0.253 | 0.174 |
| **MAPE** | 1 | 1 | 1 | 0.842 | 0.761 | 0.763 | 0.369 | 0.222 | 0.162 |

Note: The entries in this table are ratios of the RMSE and MAPE of the best linear and nonlinear models to the RMSE and MAPE of the random walk model, respectively.



1. They also prove that the bootstrap predictor is asymptotically unbiased. [↑](#footnote-ref-1)
2. These findings are based on an artificial neural network (*ANN*) model and a non‐parametric kernel function to approximate the unknown DGP, respectively. [↑](#footnote-ref-2)
3. However, one may note that at least for the *LSTAR2 DGP* the rejection frequencies do not decline monotonically, but have a local maximum at *h=* 6, 7. Although this fact does not invalidate our general conclusion, it appears that the local maximum is more pronounced as  increases. Thus, this finding may simply be due to the more pronounced nonlinear properties induced by an increasing*θ*. [↑](#footnote-ref-3)
4. For instance, among the countries for which the null of linearity cannot be rejected, 63%, 70%, and 60% of the PPI-based, CPI-based RERs and REERs, respectively belong to countries from the Euro-zone. [↑](#footnote-ref-4)
5. When comparing forecast horizons greater than one, it appears that as the forecast horizon increases the degree of nonlinearity becomes less pronounced (i.e., for the REER case, the degree of nonlinearity is significant at the 10% level in most of the instances for the twelfth forecast horizon; however for the eighth forecast horizon most of the instances where there appears to be nonlinear behavior this occurs at the 5% or 1% level, respectively). [↑](#footnote-ref-5)
6. We make sure each forecast series has the same number of observations by adjusting the sample size to cover the same time period. [↑](#footnote-ref-6)
7. For example, Piet and Van Hulle (1995), Enders and Falk (1998), Sarantis, N. (1999), Kilian and Taylor (2003), Sollis, Leybourne, and Newbold (2002), Taylor and Sarno (2002), Imbs, et al. (2003), Bec and Carrasco (2004), and Shintani, Terada-Hagiwara, and Yabu (2013) all consider the random walk model a benchmark by which to judge an exchange rate model. Levich and Poti (2014) paper is an exception in that it seeks to ascertain periods in which it was possible to make an expected speculative profit.

 [↑](#footnote-ref-7)